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Belief Reliability Analysis of Traffic Network: An Uncertain Percolation Semi-Markov Model

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Abstract

Traffic reliability is a crucial property of the transportation system, showing its ability to resist traffic jams or collapse. Traditional traffic reliability analysis only considers stochastic uncertainty but neglects epistemic uncertainty, which widely exists in the traffic network and leads to an underestimation of traffic failure. In this paper, we introduce uncertainty theory to model epistemic uncertainty, thereby developing a belief reliability analysis method for transportation systems based on the traffic performance margin. We established an uncertain percolation semi Markov (UPSM) model to describe the essential physical characteristics of the traffic accidents considering both stochastic and epistemic uncertainty. And the uncertain percolation model is utilized to describe the traffic performance degradation and the semi Markov process is developed to represent the influence of random emergency events. According to the traffic failure propagation process, a simulation method for calculating belief reliability is proposed. Finally, a case study was given to illustrate the proposed method.

1. Introduction

Nowadays, The thorny traffic congestion has become a key factor restricting the rapid development of cities, prompting scholars to study the traffic problems. Then, traffic reliability

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theory has been gradually developed. The study of traffic reliability began in 1980, which was originally defined as the capacity of urban transport network. At present, traffic reliability has become an important indicator to measure the service quality of traffic network and evaluate the operation of urban transportation system.

The traffic reliability has been analyzed from various perspectives. Mine and Kawai [1] first proposed the node connection reliability as a measure of the probability of maintaining connection between different nodes in transportation system. For travelers, the most direct indicator of road smoothness is travel time, because of which scholars have started focusing on the research on reliability of travel time. Asakura and Kashiwadani [2] put forward the concept of travel time reliability for the first time. Then, Anthony Chen [3] proposed the concept of traffic network capacity reliability based on the connected reliability and travel time reliability. In addition, they also established a capacity reliability model and solved it by the Monte-Carlo technique. It could be seen that almost all of the above research was based on Traffic Flow Theory. In recent years, Li [4] started began to study traffic reliability with statistical physics method. He proposed a reliability model of large-scale transportation network based on the Percolation Theory, which regarded the propagation of traffic congestion as a percolation process. Moreover, he also analyzed the state of the traffic network subgroup, and used the percolation threshold as the network failure criterion.

Traditional traffic network reliability methods are mostly based on the probability method. In the perspective of uncertainty, these methods only consider the stochastic uncertainty describing the internal randomness of the objective world. However, the traffic network shows typical nonlinear, strong coupling and pan-space-time characteristics. Considering these essential characteristics are multidimensional, uncertain and dynamic, there must be epistemic uncertainty in the transportation network. The so-called epistemic uncertainty is caused by people's incomplete understanding of the real world. In a word, the stochastic uncertainty and epistemic uncertainty affect the system at the same time. Ignoring the epistemic uncertainty in traffic network reliability analysis may lead to inaccurate or underestimated traffic reliability. Therefore, in the reliability analysis of traffic networks, both of these uncertainties need to be considered.

A good choice to deal with this issue is to use belief reliability theory, which is a new reliability theory considering both Stochastic and epistemic uncertainty [5,6]. Belief reliability theory introduces two new mathematical theories, namely uncertainty theory and chance theory into system reliability. Uncertainty theory is an axiomatic mathematical system founded and refined by Liu [7,8], and has been widely used in various areas to model epistemic uncertainty, including uncertain finance [9,10], decision making [11,12], uncertain control [13], maintenance optimization [14], etc. Chance theory was put forward by Liu in 2013. It combines probability theory with uncertainty theory, which makesthe belief reliability theory can simulate the stochastic and epistemic uncertainty of systems. On the basis of the strong mathematical theories, belief reliability has made great progress in recent years. Zeng et al. [15,16] first studied the belief reliability of uncertain systems under the framework of uncertainty theory and put forward a belief reliability analysis method based on cut sets. Wen and Kang [17] measured the belief reliability of Boolean uncertain random systems by chance theory, and gave several belief reliability formulas. Then, Zhang [18] et al. generalized the definition of belief reliability more concretely as the chance that the system state is within the feasible domain. They also discussed two conditions of belief reliability degradation, and showed the connotation of belief reliability from three aspects: failure time, performance margin and function level. Zeng et al. [19] developed an evaluation method for component belief reliability using performance margin model by representing epistemic uncertainty as adjustment factors to the Stochastic uncertainty factors. In 2020, Kang [20] proposed the theoretical framework of belief reliability and the basic method after summarizing the research results of his team, which made belief reliability a real science.

However, in literature, there are few researches on the belief reliability analysis methods of traffic network so far. To address the issue, this paper proposes a new method to measure and analyze the belief reliability of traffic network based on the concept of performance margin, and propose an uncertain percolation semi-Markov (UPSM) model to describe the failure propagation process. In this model, the relative size of maximal connected clique G is regarded as the critical performance parameter. Then, the performance margin can be obtained by considering the epistemic uncertainty in the percolation process and the Stochastic uncertainty in emergency. Finally, the belief reliability can be calculated by using a simulation method.

The rest of the paper is organized as follows. Sect. 2 offers a brief overview of some basic concepts and properties about uncertainty theory and belief reliability theory. In Sect. 3, we make an uncertain analysis of traffic network, based on the performance parameter and performance margin which have been obtained before. The uncertain percolation semi-Markov(UPSM) model is constructed in Sect. 4. We analyzed the belief reliability by simulation method in Sect. 5. In Sect. 6, a case study about the UPSM is performed to illustrate the model. Finally, Sect. 7 draws the general conclusions.

2. Preliminaries

In this section, some basic knowledge of uncertainty theory, belief reliability theory and semi-Markov model will be introduced, preparing for the establishment of the belief reliability model of traffic network afterwards.

2.1. Uncertainty Theory

Uncertainty theory is a new mathematical theory parallel to probability theory which founded by Liu [7] in 2007 and refined by Liu [8] in 2010. In uncertainty theory, belief degrees of events are quantified by defining uncertain measures:

DefinitionII.1 (Uncertain Measure [7]). Let Γ be a nonempty set, and $\mathcal{L}$ be a σ-algebra over Γ. A set function $\mathcal{M}$ is called an uncertain measure if it satisfies the following four axioms:

Axiom 1. (Normality Axiom). $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2. (Duality Axiom). $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^C\} = 1$ for any event $\Lambda \in \mathcal{L}$.

Axiom 3. (Subadditivity Axiom). For every countable sequence of events Λ_1,Λ_2,..., we have

$$\mathcal{M}\left\{\cup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}. \tag{1}$$

Axiom 4. (Product Axiom [21]). Let $(\Gamma, \mathcal{L}, \mathcal{M})$, $k = 1, 2, \ldots$ be uncertainty space. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$M\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \overset{\infty}{\underset{k=1}{min}}\,\mathcal{M}_k\{\Lambda_k\}. \tag{2}$$

DefinitionII.2 (Uncertain Variable [7]). An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set $\mathcal{B}$ of real numbers.

DefinitionII.3 (Uncertain Distribution [21]). The uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x) = \mathcal{M}\{\xi \le x\}$ for any real number x.

The calculation rule in the uncertainty theory is very different from probability theory, for which inverse distributions play a central role in uncertainty theory. Inverse distributions are the fundamentals of calculating between uncertain variables.

DefinitionII.4 (Inverse Uncertain Distribution). Let ξ be a uncertain variable with canonical uncertainty distribution $\Phi(x)$, the inverse distribution $\Phi^{-1}(x)$ of $\Phi(x)$ is defined as the inverse uncertain distribution of .

Theorem 1. (Operational law [21]). Assume that uncertain variable ξ have an inverse uncertain distribution $\Phi^{-1}(\alpha)$. $\xi_1, \xi_2, \ldots, \xi_n$ is a series of uncertain variables with inverse uncertain distributions $\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)$, and they satisfied

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n). \tag{3}$$

If ξ is monotonically increasing with respect to $\xi_1, \xi_2, \ldots, \xi_i$, and decreasing with respect to $\xi_{i+1}, \xi_{i+2}, \ldots, \xi_n$, their inverse uncertain distributions satisfied

$$\Phi^{-1}(\alpha) = f\big(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_i^{-1}(\alpha), \Phi_{i+1}^{-1}(1-\alpha), \Phi_{i+2}^{-1}(1-\alpha), \ldots, (1-\alpha)\big). \tag{4}$$

2.2. Belief Reliability Theory

Belief reliability theory was founded by Kang [20], to make up for the vulnerability of traditional reliability analysis method that ignores the Epistemic uncertainty. The related definitions are shown below.

DefinitionII.5 (Belief reliability [20]). Let the system state variable ξ be an uncertain random variable, and Ξ be the feasible domain of the system state. Then belief reliability is defined as the chance that system state is in the feasible domain, i.e.,

$$R_B = Ch\{\xi \in \Xi\}. \tag{5}$$

RemarkII.1. If the state variable ξ degenerate to a random variable, the belief reliability metric will be a probability. Let $R_B(P)$ denotes the belief reliability under probability theory. Then

$$R_B = R_B(P) = Pr\{\xi \in \Xi\}. \tag{6}$$

is means that the system is mainly influenced by Stochastic uncertainty, and the belief reliability degenerate to the probability theory-based reliability metric.

RemarkII.2. If the state variable ξ degenerate to an uncertain variable, the belief reliability metric will be a belief degree. Let $R_B{}^{(U)}$ denotes the belief reliability under uncertainty theory. Then

$$R_B = R_B{}^{(U)} = M\{\xi \in \Xi\}. \tag{7}$$

This means that the system is mainly influenced by epistemic uncertainty, and the belief reliability degenerate to the uncertainty theory-based reliability metric.

DefinitionII.6 (Performance Parameter and Fault Criterion [20]). The completion of product functions can be characterized by the parameter p and its corresponding p_{th}. When p

exceeds the limit range of p_{th} the product will fail, the parameter p is defined as the performance parameter and the parameter p_{th} is defined as the fault criterion corresponding to the performance parameter p.

RemarkII.3. According to the limited form of performance parameter p and fault criterion p_{th}, performance parameters can be divided into three categories:

(1) Smaller-the-better(STB): when and only when $p \geq p_{th}$ the product failure, the performance parameter is defined as smaller-the-better.
(2) Larger-the-better(LTB): when and only when $p \leq p_{th}$ the product failure, the performance parameter is defined as large-the-better.
(3) Nominal-the-better(NTB): when and only when $p \leq p_{th,L}$ or $p \geq p_{th,U}$ the product failure, the performance parameter is defined as nominal -the-better.

DefinitionII.7 (Performance Margin [20]). Let p be the performance parameter of product, p_{th} be the fault criterion corresponding to the performance parameter p. It is defined that

$$m = \begin{cases} p_{th} - p & if\ p\ is\ STB \\ p - p_{th} & if\ p\ is\ LTB \\ \min\left(p_{th,U} - p, p - p_{th,L}\right) & if\ p\ is\ NTB \end{cases}, \tag{8}$$

is the performance margin corresponding to p.

RemarkII.4. Let the performance margin as the system state variable, the belief reliability can be calculated by $Ch\{m > 0\}$.

2.3. Semi-Markov Model

We use $S(t)$ to represent the state of the system. The system time points are denoted as $t_1, t_2, t_3, \ldots t_m, \ldots, t_n$ which are easy to observe and record the state o the system. The corresponding system state can be written as $S(t_1), S(t_2), S(t_3), \ldots S(t_n)$. Suppose that the two states of the system are the i state and the j state, and the one-step transition probability between any two system states is written as P_{ij}, the residence time of the state is written as T_{ij}, the residence time distribution function is written as $F_{ij}(t)$ whose probability density function is written as $f_{ij}(t)$, and the state transition process of a system is expressed as follows:

$$P_{ij} = \Pr\left\{S(t_n) = s_j | S(t_{n-1}) = s_i\right\}, \quad i, j \in \{1, \ldots, n\}. \tag{9}$$

$$F_{ij}(t) = \Pr\left\{T_{ij} \leq t\right\} = \int_{-\infty}^{t} f_{ij}(t) du. \tag{10}$$

If T_{ij} can obey any distribution function, that is, $F_{ij}(t)$ has any form, then the current state transition process can be called a semi-Markov process. If $F_{ij}(t)$ only obeys exponential distribution, then the current state process is Markov process. In this way, Markov process belongs to a special form of semi-Markov process.

For quantifying various potential associations in deconstructing semi-Markov processes, the engineering field usually uses kernel matrix, which is denoted as $K_{ij}(t)$ representing the one-step transition probability of the system from state i to state j in time t, and

$$K_{ij}(t) = \Pr\left\{S(t_n) = s_j, t_n - t_{n-1} \leq t | S(t_{n-1}) = s_i\right\}, \quad i, j \in \{1, \ldots, n\}. \tag{11}$$

$$P_{ij} = \lim_{t\to\infty} K_{ij}(t). \tag{12}$$

$$\sum_{j=1}^{n} P_{ij} = 1 \tag{13}$$

DefinitionII.7 (kernel matrix [22]). Every n × n matrix K of non-decreasing functions null on R+ satisfying properties Eq.(12) and Eq.(13) is called a Semi-Markov matrix or a Semi-Markov kernel.

$$P_{ij} = \lim_{t\to\infty} K_{ij}(t). \tag{14}$$

$$F_{ij}(t) = \tfrac{K_{ij}(t)}{P_{ij}}. \tag{15}$$

According to the above, it can be deduced that the residence time T_i of the system, the residence time distribution function $F_i(t)$ and the probability density function corresponding to $f_i(t)$ are as follows

$$T_i = \sum_{i=1}^{n} P_{ij} T_{ij}. \tag{16}$$

$$F_i(t) = \sum_{i=1}^{n} K_{ij}(t) = \sum_{i=1}^{n} P_{ij} F_{ij}(t). \tag{17}$$

$$f_i(t) = \sum_{i=1}^{n} P_{ij} f_{ij}(t). \tag{18}$$

It is assumed that the system is in state i at the time t_n, and may be in state i, j or e at the time t_{n+1}, then [23]

$$K_{ii}(t) = \int_0^t \big(1 - F_{ij}(u)\big)(1 - F_{ie}(u))dF_{ii}(u). \tag{19}$$

$$K_{ij}(t) = \int_0^t (1 - F_{ii}(u))(1 - F_{ie}(u))dF_{ij}(u). \tag{20}$$

$$K_{ie}(t) = \int_0^t (1 - F_{ii}(u))\big(1 - F_{ij}(u)\big)dF_{ie}(u). \tag{21}$$

Kernel matrix is the key solution point in the semi-Markov process. The residence time distribution function of the system in each state can be calculated by constructing kernel matrix.

3. Belief Reliability Analysis of The Traffic Network

This section analyzes the uncertainty of the traffic network, based on which we present the performance parameter and performance margin of the traffic network. Sect.3.1 makes uncertainty analysis of the traffic network. The performance parameter and performance margin are given in Sect.3.2.

3.1. Uncertainty Analysis of The Traffic Network

There is a great deal of uncertainty in the traffic network. The percolation process of traffic network itself and the influence caused by traffic light emergency are the main focus of our analysis.

Capacity variation is the most important factor to determine the percolation process. Factors influencing intersection capacity are as follows. The main influencing factors can be divided into five categories [24]: road conditions, traffic conditions, traffic demand, individual selection behavior, and running state. Road conditions include road length and width, intersection spacing and road grade, which are determined or a given traffic network at a given time. So the road conditions can be described as random variable. The traffic conditions include the road vehicle type constitution and the traffic mode. Even for a certain road, the type of vehicle in it is currently undetectable and uncertain, therefore the traffic conditions are described as uncertain variable. The matching degree of traffic demand and road network can be calculated by matching algorithm which is described as random variable. The individual choice behavior of traffic network is related to the choice made by each traveler. The traveler's behavior pattern involves traffic human factors and is affected by the weather and the driver's psychological pattern, which is described as uncertain variable. The running state of the road includes unimpeded, congested or intermediate state. Now the definition of different states is determined by the relationship between the average speed and the speed threshold. However, the speed threshold should not be described as random variable because of the different traffic environment which is also a shortcoming of the current traffic percolation theory. Taking the velocity threshold as a certain value makes the evaluation of the state of each road not accurate. Based on what has been discussed above, the uncertainty theory is chosen to describe the performance degradation process affected by the capacity.

Due to the traffic lights strictly comply with national standards, there is plenty of data to describe their fault rate through a large number of reliability test. The traffic lights may transfer between different states. Generally speaking, the performance of traffic lights will decline with time, and the transition time of state will be affected by the use time of traffic lights which makes the traffic lights not follow the Markov process. Therefore, the Semi-Markov model is chosen to build the model of performance degradation process affected by the traffic light emergency.

3.2. Performance Margin Modeling and Belief Reliability Model

The relative size of maximal connected clique [25,26] G is chosen to delegate the global connectivity degree of the traffic network. Thus, the performance parameter of the traffic network can be represented by the relative size of maximal connected clique G. G is evaluated with

$$G = \frac{N'}{N}, \tag{22}$$

where the N' is the number of nodes of maximal connected clique in the current network, and N is the number of nodes in the original network.

We define the G_{th} as the number of relative size of maximal connected clique threshold. G is Greater-the-better (GTB) [21] because that when and only when $G \leq G_{th}$ the product failure. Then, the performance margin m can be defined based on the performance parameter and the threshold G_{th},

$$m = G - G_{th}. \tag{23}$$

Thus, the belief reliability can be defined according to belief reliability theory,

$$R = Ch(m \geq 0) = Ch(G \geq G_{th}). \tag{24}$$

In order to facilitate calculation, Eq.(11) can written as:

$$R = Ch(m \geq 0) = Ch(G \geq G_{th}) = Ch\left(\frac{N`}{N} \geq \frac{N`_{th}}{N}\right), \tag{25}$$

4. UPSM Model for Traffic Network Failure Propagation

4.1. Uncertain Percolation Model

From the perspective of statistical physics, the overload model of traffic network based on uncertainty theory and Semi-Markov model is used to represent the failure propagation of traffic network based on percolation theory.

Traffic network failure propagation model is based on the performance degradation process of the nodes, which is dived into two parts in this paper: the performance degradation process affected by the capacity and performance degradation process affected by emergency. We take the impact of capacity into consideration in the beginning.

The traffic network failure propagation model and the solving process of G are shown as below.

Firstly, the topology of the traffic network is represented. Assuming a traffic network with m roads and n intersections, the roads of which are represented as the lines, and the intersections are replaced by nodes. The traffic network is seen as an uncertain random system which is influenced by both Stochastic and epistemic uncertainty.

Subsequently, we define $x_j(t)$ as the state variable of line $j(j \in [1, m])$ and it is an uncertainty measure that average velocity of line j greater than velocity threshold. Hence, $x_j(t)$ is evaluated with

$$x_j(t) = M\left(v_j(t) > \theta\right), \tag{26}$$

where $v_j(t)$ is the average velocity of the road j and θ is the velocity threshold.

$v_j(t)$ is a constant for a certain traffic network at certain time point. According to the Sect.3.1, θ is influenced by all five categories. Therefore, θ is seen as an uncertain variable, whose uncertainty distribution can be evaluated by the empirical distribution.

We can define the state variables of the nodes after obtaining $x_j(t)$. This paper defines $L_i(t)$ as the sate variable of intersection i. $L_i(t)$ can be regarded as the load of the intersection i, which reflects the capacity of the intersection load. Assuming that all vehicles choose the shortest path, and the capacity of load can be described as betweenness. $x_j(t)$ is an uncertain random variable which ranges from 0 to 1 excluding 0 and 1, because the state variable of line is calculated by uncertainty theory. Thus, the new definition of the weight of line and betweenness is given in an uncertain random system.

DefinitionIII.1 (Existence Risk): Let $x_j(t)$ be the state variable of the j_{th} line in an uncertain random network, the existence risk of j_{th} road is defined as

$$W(j) = 1 - x_j(t). \tag{27}$$

RemarkIII.1. If a path contains multiple roads, the existence risk of the path is recalculated as the sum of all the existence risk of roads, as shown in the Eq(15):

$$\sigma_{st} = \sum_{st} W(j). \tag{28}$$

where σ_{st} means the number of least risk paths starting and ending with node s and node t.

DefinitionIII.2 (Betweenness Centrality in Uncertain Random Network): The betweenness centrality of the node i in uncertain random network is defined as

$$C_B(i) = \sum_{s \neq i \neq t \in V} \frac{\sigma_{st}(i)}{\sigma_{st}}, \tag{29}$$

Where $\sigma_{st}(i)$ means the number of least risk paths through node i starting and ending with node s and node t.

In ordinary networks, the betweenness centrality of node usually indicates the importance of a node as a connecting point, as well as the capacity characteristics of node. Here, we also use the betweenness centrality of the uncertain random network to represent the node capacity in the uncertain random network. Therefore, the $L_i(t)$ can be evaluated with

$$L_i(t) = C_B(i) = \sum_{s \neq j \neq t \in V} \frac{\sigma_{st}(i)}{\sigma_{st}}. \tag{30}$$

We define the tolerance parameter α and initial load $L_0(t)$ to measure the overload criterion. $L_0(t)$ means the initial load condition of every node in traffic network on moment t. The α means the capacity of overload of node, which can fail if the real-time load is greater than the load limit. Thus, the overload criterion is given with

$$M\{L_i(t) - L_0(t) \cdot \alpha > 0\} \geq 0.95, \tag{31}$$

where the α is affected by road conditions, traffic conditions and traffic demand. Thus, the α should be regarded as an uncertain variable.

We can run the overload model based on the definitions after this series of tasks. For the traffic network, the full overload model is demonstrated below. The topology of the traffic network has been obtained, each node of which is then given its initial load $L_0(t)$. The velocity threshold is evaluated based on the analysis of the line to determine the state variable. Then we calculate the betweenness of nodes according to the state variable of the lines. The tolerance parameter is given by analyzing each node to acquire the overload criterion. Next, each node is judged by overload criterion. If the node exceeds the overload criterion, it is considered to be invalid and then the node will be removed. According to the percolation theory [27], the edge connected with the node is deleted when removing the node, changing the topology of the network, as shown in the figure 1.

According to the new topology, the betweenness of nodes is recalculated, which means load redistribution. The whole overload process doesn't end until all nodes' load meet the overload criterion, and then traffic network reach the equilibrium state. The topology of equilibrium state determines the condition of maximal connected clique, and then the number of nodes of maximal connected clique N‘ is obtained.

4.2. Semi Markov Model for The Impact of Emergency

In the real traffic network, there are some occasional failures of traffic light such as unresponsive light, wrong display and permanently flashed yellow light. These failure modes

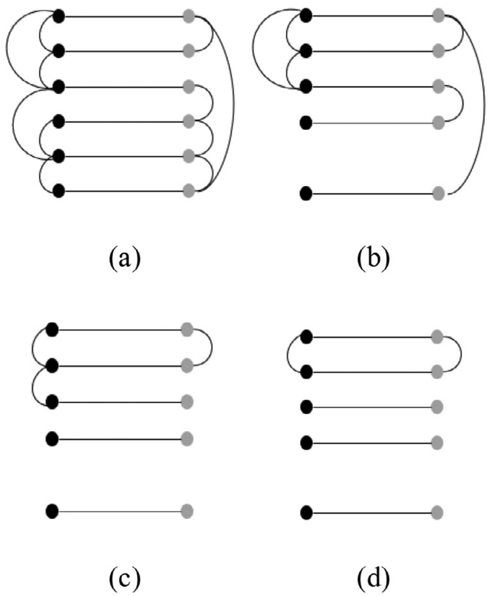

Figure 1. Percolation process

will affect vehicle operation rules of the previous route, and then lead to overload of network nodes, traffic congestion, and even lead to cascading failure of road network. This section considered the process of road network performance degradation caused by sudden failures of traffic lights.

The failure process of the traffic light, that is, the transition from normal to failure is actually the process of system transition between different states. Therefore, the application of Markov model to establish the state transition process is consistent with the current situation. The transition time and the state transition probability of pure Markov process are not affected by the past state. They are memoryless and only determined by the current state. Therefore, the state transition time of this process obeys an exponential distribution. For practical situations, Markov processes have too many limitations, while semi-Markov process can obey more distribution forms, such as Weibull distribution, due to its residence time. Therefore, using semi-Markov model makes the system to be analyzed have more choices, and the analysis results are more realistic. Therefore, this section uses semi-Markov model to describe the process.

At this point, we can consider the impact of an emergency on performance degradation. There are all sorts of emergencies in the traffic network, and here we choose the traffic light failure.

Traffic light failure is a random process, the probability of which depends on the reliability of traffic light products. The Semi-Markov model is chosen to describe the traffic light failure, which divides the traffic light status into three states: normal, failure A, failure B. The state sets $S = \{S_1 = normal, S_2 = warining,\ S_3 = failure\}$. The definitions of the three states are given below.

DefinationIII.3:

State ‘normal’: If the traffic light works normally, and the red, yellow and green flashing alternately, $S = S_1$.

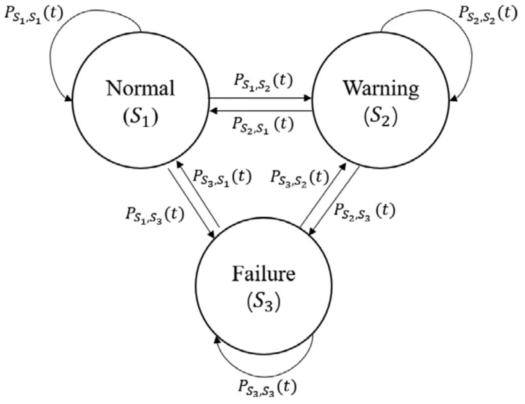

Figure 2. The schematic diagram of traffic light semi-Markov model

State ‘warning’: If the traffic light keeps yellow shining, $S = S_2$.

State ‘failure’: If the traffic light is out of order, including keeping black whole or part and physical damage, $S = S_3$.

Under the Semi-Markov model of traffic network, the characteristics of traffic light nodes under the three states and the relationship between the three states can be studied respectively. Figure 2 shows the schematic diagram of traffic light semi-Markov model.

$P_{S_i,S_r}(t)$ represents the probability of going from state S_i to state S_r. When the current state is S_i and the next state is S_r, the time distribution from S_i to S_r follows $F_{i,r}(t)$. Thus, the present state is not only related to the past state, but also related to how long the system stayed in the past state and the present state under the semi-Markov model of traffic light failure.

Different state of traffic lights have different effects on nodes and edges. If the traffic light corresponding to node i is in state S_1, the node i and the edges linking to node i will be only influenced by capacity of node i. If the traffic light corresponding to node i is in state S_2, it indicates that the traffic volume at this node is small, which only needs the attention of the vehicle drivers instead of the traffic lights to help adjust the flow. Therefore, the performance degradation caused by capacity and traffic lights on this node can be ignored. If the traffic light corresponding to node i is in state S_3, it can be assumed that both the node and its linked edges are invalid. The performance degradation caused by traffic lights is shown in the figure 3.

4.3. Belief Reliability Solving Process with Simulation Method

R maintains the form of chance measure when $N`_{th}$ is a constant for a certain traffic network and it can be only obtained by simulation at present. If R degrades into uncertainty measure, it can be obtained by the distribution of G and G_{th}, which denote as $\psi(t)$ and $\omega(t)$ respectively.

By calculating the corresponding G values of different α and θ, we can get the function expression of G with respect to α and θ.

$$G = g(\alpha, \theta). \tag{32}$$

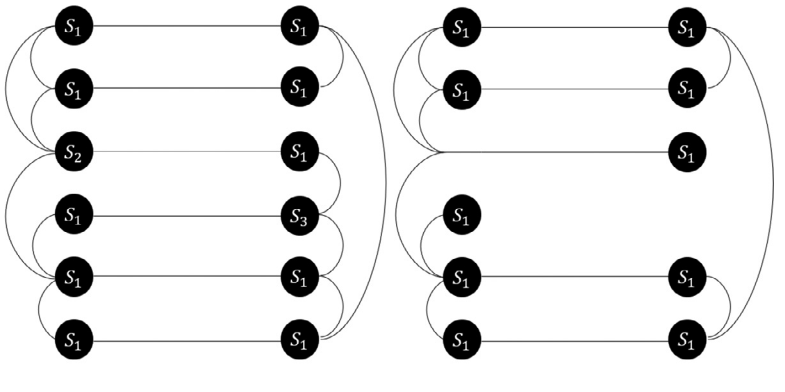

Figure 3. The schematic diagram of performance degradation caused by traffic light

From what has been discussed above, the model flow chart about function form of G is shown as figure 4.

$\psi(t)$ and $\omega(t)$ have to be solved out firstly. The latter can be obtained by the statistical data of traffic network, but the former can only be obtained by the distribution of θ and α. Assuming that $\phi_1(t)$ is the distribution of q, and $\phi_2(t)$ is the distribution of α. If the G is strictly increasing with respect of q and α, the inverse distribution $\psi^{-1}(a)$ of G is

$$\psi^{-1}(a) = g\left(\phi_1{}^{-1}(a), \phi_2{}^{-1}(a)\right). \tag{33}$$

If G is strictly decreasing with respect of θ and α, the inverse distribution $\psi^{-1}(a)$ of G is

$$\psi^{-1}(a) = g\left(\phi_1{}^{-1}(1-a), \phi_2{}^{-1}(1-a)\right). \tag{34}$$

If G is strictly increasing with respect of θ and strictly decreasing with respect of α, the inverse distribution $\psi^{-1}(a)$ of G is

$$\psi^{-1}(a) = g\left(\phi_1{}^{-1}(a), \phi_2{}^{-1}(1-a)\right). \tag{35}$$

If G is strictly increasing with respect of α and strictly decreasing with respect of θ, the inverse distribution $\psi^{-1}(a)$ of G is

$$\psi^{-1}(a) = g\left(\phi_1{}^{-1}(1-a), \phi_2{}^{-1}(a)\right). \tag{36}$$

Then, we get the inverse function of $\psi^{-1}(a)$, that is the distribution of G. Thus, we have

$$R = \sup_{t \in R}(1 - \psi(t) \wedge \omega(t)). \tag{37}$$

G_{th} is a constant under special circumstance, we have

$$R = M(G \geq G_{th}) = 1 - M(G \leq G_{th}). \tag{38}$$

Here we introduce the theorem about uncertainty theory to help derive the result.

TheoremIII.1 [28]**. Let** ξ be an uncertain variable with inverse uncertainty distribution $\psi^{-1}(\beta)$, and let β and c be constants with $0 < \beta < 1$. Then,

$$M\{\xi \leq c\} \geq \beta, \tag{39}$$

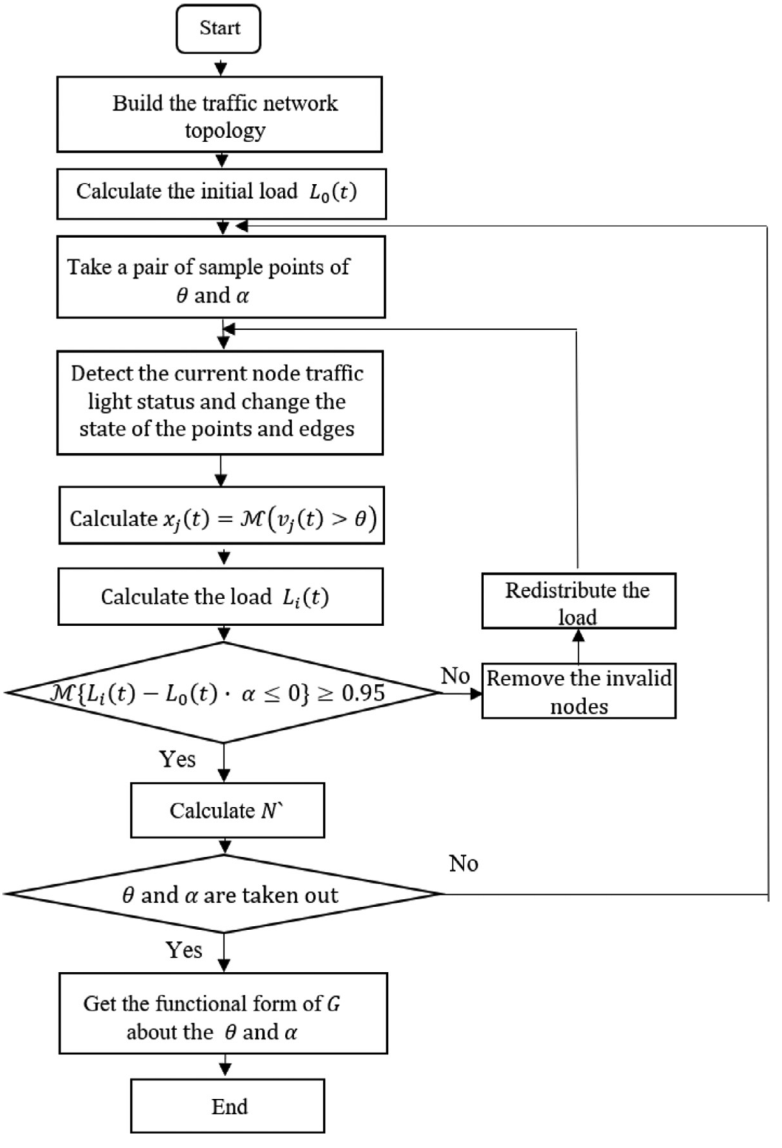

Figure 4. The model flow chart about function form of G

if and only if

$$\psi^{-1}(\beta) \leq c. \tag{40}$$

According to TheoremIII.1, we have

$$G_{th} \geq \psi^{-1}(1 - R), \tag{41}$$

from which we can solve out the R.

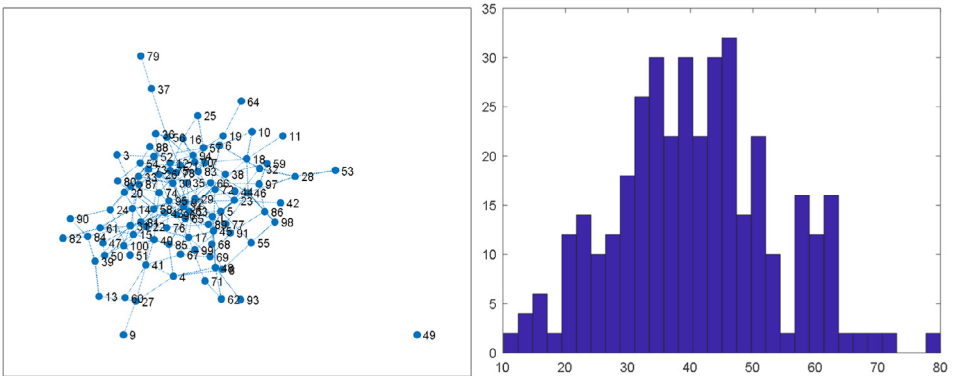

Figure 5. The abstract network and scatter gram of average velocity

Table 1
The table of cumulative distribution function of conditional sojourn time

i	r	$F_{i,r}(t)$	**parameter choice**
1	2	$1-e^{-\lambda_{12}\cdot t}$	$\lambda_{12}=8\times10^{-4}$
1	3	$1-e^{-\lambda_{13}\cdot t}$	$\lambda_{13}=6\times10^{-6}$
2	1	$\Phi(\frac{t-\mu_{21}}{\sigma_{21}})$	$\mu_{21}=500h,\ \sigma_{21}=64h$
2	3	$1-e^{-\lambda_{23}\cdot t}$	$\lambda_{23}=6\times10^{-6}$
3	1	$\Phi(\frac{t-\mu_{31}}{\sigma_{31}})$	$\mu_{31}=1200h,\ \sigma_{31}=150h$
3	2	$\Phi(\frac{t-\mu_{32}}{\sigma_{32}})$	$\mu_{32}=1200h,\ \sigma_{32}=150h$

5. Case Study

5.1. Background and Assumption

In order to verify the validity of this model, we build a traffic network which owns 100 crossroads and 347 roads to analyze the reliability. We treat it as an abstract network with 100 nodes and 347 lines. The average velocity of each line at time t are recorded by the adjacency matrix. The abstract network and scatter gram of average velocity are shown as figure 5.

For traffic light, the table of conditional distribution probability functions for transition intervals is shown as table 1.

5.2. Belief Reliability Calculation

We calculate G under different θ and α and the result is shown as figure 6,

At a certain interval of θ value, G will produce a drastic mutation, and the whole surface presents a geometric form of S mixed slope. According to the least nearest neighbor method, the function that fits the scatter graph best is:

$$G=-0.05+\frac{1}{1+e^{-2\alpha+0.1\theta}}. \tag{42}$$

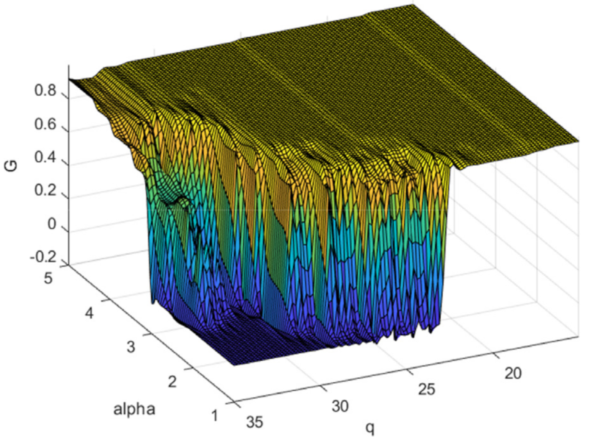

Figure 6. The figure of G value about different θ and α

The probability distribution of G can be obtained by counting the value of its function, which is

$$\varphi(a) = 0.11 \cdot \tan(2.67a - 1.05) + 0.12. \tag{43}$$

Then, the uncertainty distribution of G can be obtained by the uncertainty distribution ϕ_1 of q and ϕ_2 of α. We set up expert questionnaires and use expert experience data to determine their uncertainty distribution.

$$\phi_1(a) = \left(1 + \exp\left(\frac{\pi(22.5-a)}{\sqrt{3}\times 7.5}\right)\right)^{-1}. \tag{44}$$

$$\phi_2(a) = \left(1 + \exp\left(\frac{\pi(1-\ln(a))}{\sqrt{3}\times 0.3}\right)\right)^{-1}. \tag{45}$$

From the surface diagram, it can be seen that G monotonically rises with respect to α, and monotonically falls with respect to θ. Thus, the inverse distribution of G can be written as Eq.(33):

$$\begin{aligned}\psi^{-1}(a) &= f\left(\phi_2{}^{-1}(a), \phi_1{}^{-1}(1-a)\right) = -0.05 + \frac{1}{1 + e^{-2\phi_2{}^{-1}(a) + 0.1\phi_1{}^{-1}(1-a)}} \\ &= -0.05 + \frac{1}{1 + e^{-2\cdot e^{1-\frac{3\sqrt{3}\cdot\ln\left(\frac{1}{a}-1\right)}{10\cdot\pi}} + 0.1\cdot\left(22.5 - \frac{15\sqrt{3}\cdot\ln\left(\frac{1}{1-a}-1\right)}{2\cdot\pi}\right)}}.\end{aligned} \tag{46}$$

Hence one can see that the condition from disconnected to connected of the traffic network is abrupt, and the value of G_{th} is a fixed constant for a specific traffic network. G_{th} of the traffic network in this case is 0.92. Thus, we can press the R as:

$$R = 1 - a = 0.6338. \tag{47}$$

which is the value of belief reliability of traffic network at moment t.

6. Conclusion

In this paper, we have proposed a newly emerging traffic network belief reliability evaluating model based on belief reliability theory and semi-Markov model. From the view of theoretical completeness and engineering practicality, this paper has developed a new calculation method taking both Stochastic and epistemic uncertainty into consideration. Based on uncertainty analysis of traffic network, the relative size of maximal connected clique G is selected as the performance parameter and the relevant performance margin is obtained. We divide the performance degradation process of traffic network into two parts for analysis, namely, the performance degradation of the node itself due to the influence of the percolation process and there duction of the traffic light incidence. Combining them into the solution of the traffic network performance parameters, the uncertain percolation semi-Markov(UPSM) model is constructed, and the traffic network is constructed to describe the failure propagation process from the beginning to the steady state. This provides a wonderful idea for considering the influence of multiple factors in the model, and has significance reference value. In addition, this paper managed to propose a traffic network belief reliability calculation method. In this case, based on the UPSM model, the equilibrium state of the traffic network is determined and the performance parameters are obtained, so that a belief reliability analysis can be carried out at any time. As a pioneering research on the reliability of transportation network, this paper put forward a reliability measurement method considering both epistemic uncertainty and Stochastic uncertainty, and establishes a theoretical foundation for improving the reliability of traffic network.

The future work may focus on the different traffic network performance parameters such as traffic network travel time, capacity, etc. Then, we can contrast them with others to find out the most suitable performance parameter.

Declaration of Competing Interest

No conflict of interest exits in the submission of this manuscript, and manuscript is approved by all authors for publication. I would like to declare on behalf of my co-authors that the work of this paper is originally created by our team. We have not published this paper in any other journal. All the authors listed have approved the manuscript that is enclosed.

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