State Estimation for Periodic Neural Networks With Uncertain Weight Matrices and Markovian Jump Channel States

Yong Xu, Zhuo Wang, Deyin Yao, Renquan Lu, and Chun-Yi Su, *Senior Member, IEEE*

***Abstract*—This paper studies the state estimator design for periodic neural networks, where stochastic weight matrices $B(k)$ and packet dropouts are considered. The stochastic variables, which may influence each other, are introduced to describe uncertainties of weight matrices. In order to model the time-varying conditions of the communication channel, a Markov chain is employed to study the jumping cases of the stochastic properties of the packet dropouts (i.e., Bernoulli process with jumping means and variances being used to handle the packet dropouts). A state estimator is constructed such that the augmented system is stochastically stable and satisfies the H_∞ performance. The estimator parameters are derived by means of the linear matrix inequalities method. Finally, a numerical example is provided to illustrate the effectiveness of the proposed results.**

***Index Terms*—Markov chain, neural networks (NNs), packet dropouts, state estimator, stochastic parameter.**

I. Introduction

THE dynamical analysis of various neural networks (NNs) has attracted widely breadth of research interests [1]–[7]. Many remarkable theoretical works have been proposed during the past few decades, such as stability analysis for the NNs with time delays [8], [9], performance studying for the NNs with mixed time delays [10], synchronization addressing for the NNs [11], and so on. For the Markovian jump NNs with time delays, the stability and the performance were studied as well in the past decade [12]. More recently, artificial NNs have been used to deal with pattern recognition, robotics control, etc. Thus, studying on the NNs is an important practical problem, which needs more attention.

The periodic character exists in many real systems. Thus periodic systems have been widely studied in a number of fields, and many influential theoretical results, including the system analysis and synthesis [13], the optimal filter and the optimal controller design [14], the system fault detection [15], and so on, have been published for periodic systems. In [16], the state estimation problem for periodic systems with transmission delays and multiplicative noise was addressed. An efficient control design technique for the discrete-time positive periodic systems was developed [17]. Recently, the periodic scheduling method has been introduced to networked control systems to deal with the issue of the communication capacity constraints [18]. For the NNs, the periodic property also exists, and some works have been completed [19]. However, how to make full use of the periodic properties of the NNs to analyze the stability, the performance, and design the corresponding estimator become challenging problems, which have not been fully considered yet.

To know the full states of the NNs is required to analyze the biological NNs and to accomplish a specified work in the artificial NNs. Due to the technical restrictions, it is infeasible to obtain full states of the NNs directly, which motivates researchers to design the estimator for the NNs on the basis of the available measurements [20]–[22]. Recently, networked control systems, which possess the advantages of reducing the cost, facilitating the maintenance, improving the flexibility, and so on, have attracted a lot of attention in the past two decades [23]–[27]. How to use the imperfect measurements obtained via the shared networks to estimate the states of the NNs becomes a hot spot, and some related works have been published. In [28], the asynchronous H_∞ filter was designed for the Markov jump NNs with multiplicative noises on the basis of the measurement transmitted via the channel with randomly occurred quantization. In [29], the event-triggered method, which can be used to improve the use ratio of the communication channels, was employed to transmit the measurements of the NNs with multidelays, and the estimator was designed using the received information. How to further study the condition of the communication channels to improve the

Manuscript received January 19, 2017; revised April 1, 2017; accepted May 17, 2017. Date of publication June 30, 2017; date of current version October 15, 2018. This work was supported in part by the China National Funds for Distinguished Young Scientists under Grant 61425009, in part by the National Natural Science Foundation of China under Grant 61673041, Grant 61503106, and Grant U1509217, and in part by the Guangdong Province Higher Vocational Colleges and Schools Pearl River Scholar approved in 2015. This paper was recommended by Associate Editor Z. Wang. *(Corresponding author: Renquan Lu.)*

Y. Xu, D. Yao, and R. Lu are with the School of Automation, Guangdong University of Technology, Guangzhou 510006, China, and also with the Guangdong Key Laboratory of IoT Information Technology, Guangzhou 510006, China (e-mail: xuyong809@163.com; deyinyao2013@gmail.com; rqlu@gdut.edu.cn).

Z. Wang is with the School of Instrumentation Science and Optoelectronics Engineering, Beihang University, Beijing 100191, China (e-mail: zhuowang@buaa.edu.cn).

C.-Y. Su is with the School of Automation, Guangdong University of Technology, Guangzhou 510006, China, and also with the Guangdong Key Laboratory of IoT Information Technology, Guangzhou 510006, China, on leave from Concordia University, Montreal, QC H3G 1M8, Canada (e-mail: chun-yi.su@concordia.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSMC.2017.2708700

estimator performance of the NNs becomes a new challenge problem.

Parametric uncertainties, including norm bounded uncertainties, polytopic uncertainties, and stochastic uncertainties, are the important factors which impact on the stability and the performance of the systems. The reasons which attract researchers to investigate the stochastic uncertain systems are summarized as follows.

1) During the system model identification, it is too complex to yield the precise system model, and the stochastic uncertain method is generally used to simplify the system model [30], [31].
2) The systems and the measurements are usually disturbed by the stochastic multiplicative noises [32].
3) The stochastic channel uncertainties, like channel fading, are unavoidable, because the communication channels are always disturbed by the environmental noises [33].

For the NNs, the weight connections between the neurons are always influenced by the external disturbance, which motivates researchers to study the uncertainties of the weight matrices [34]. Thus, how to use the stochastic uncertain model to describe the weight connections and design a robust estimator is an important problem.

Motivated by the discussion made above, this paper studies the issue of state estimation for the periodic NNs with uncertain weight matrices and unreliable communication channel. The state estimator is designed based on the imperfect measurements. According to the linear matrix inequalities (LMIs) methods, sufficient conditions are obtained to ensure that the augmented system is stochastically stable and has an H_∞ performance index γ. Finally, the effectiveness of the achieved results is illustrated by a numerical example. The main contributions of this paper are summarized as follows.

1) The stochastic periodic matrices $B(k)$ are employed to model the uncertain weight connections among the neurons via the mean and the variance, which also can describe the influences among different connections by the covariance. Thus it is more general than the existing uncertain models [35], [36].
2) A Markov chain, whose transition probability depends both on the states of the Markov chain and the system mode (i.e., periodic Markov chain), is introduced to describe the communication channel states. Different from the existing works, Bernoulli process, whose stochastic properties depend on the channel state, is employed to deal with the packet dropouts.
3) In order to improve the performance of the estimator, the system mode and channel state are both introduced to design the estimator gains (i.e., the mode and channel state-dependent estimator is proposed for the periodic Markov jump system).

In the following, the discrete-time NNs with uncertain weight matrices and Markovian jump channel states are described in Section II. In Section III, the sufficient conditions of stochastic stability and H_∞ performance for the augmented system are derived by utilizing the LMI approach. The parameters of the estimator are designed in Section IV. An illustrative example is presented in Section V. Finally, Section VI concludes this paper.

Notations: The space of n-dimensional vector, the set of $m \times n$ real matrices, and the infinite sequence of square summable are denoted by $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$, and $l_2[0, +\infty)$, respectively. The diagonal matrix is denoted by diag$\{\cdot\}$, $I_n \in \mathbb{R}^{n\times n}$ stands for the identity matrix. For a matrix, the term induced by symmetry is represented as $*$. Superscript T stands for the transposition of a matrix. For a stochastic variable $\zeta(k)$, its probability and expectation are denoted by $\mathbb{P}\{\zeta(k)\}$ and $\mathbb{E}\{\zeta(k)\}$, respectively. The covariance of variables $\zeta_1(k)$ and $\zeta_2(k)$ is denoted by $\mathbb{C}\{\zeta_1(k), \zeta_2(k)\}$.

II. System Description and Preliminaries

A. System Description

The discrete-time periodic NNs with n neurons are expressed as

$$x_i(k+1) = a_{i,\theta(k)}x_i(k) + \sum_{j=1}^{n} b_{ij,\theta(k)}(k)g_j\big(x_j(k)\big) + e_{i,\theta(k)}w_i(k), \quad i = 1, 2, \ldots, n \tag{1}$$

where $x_i(k) \in \mathbb{R}$ means the state variable of neuron i, $w_i(k)$ stands for the external disturbance belonging to $l_2 \in [0, \infty)$. The m-periodic scalar $\theta(k)$ is defined as

$$\theta(k) \triangleq \mathrm{mod}(k, m) + 1 \in \mathcal{N}_m \triangleq \{1, 2, \ldots, m\}.$$

For the periodic NNs (1), m-periodic constants $a_{i,\theta(k)}$ and $e_{i,\theta(k)}$ are known. $b_{ij,\theta(k)}(k)$ is the interconnection strength between neurons i and j, which is a stochastic parameter with the following properties [31]:

$$\begin{aligned} \mathbb{E}\big\{b_{ij,\theta(k)}(k)\big\} &= \bar{b}_{ij,\theta(k)} \\ \mathbb{C}\big\{b_{ij,\theta(k)}, b_{\alpha\beta,\theta(k)}\big\} &= \varrho_{ij\alpha\beta,\theta(k)}. \end{aligned} \tag{2}$$

The stochastic variable can be further rewritten as

$$b_{ij,\theta(k)}(k) = \bar{b}_{ij,\theta(k)} + \check{b}_{ij,\theta(k)}(k).$$

The neuron activation function $g_i(\cdot)$ satisfies $g_i(0) = 0$ and the following condition [37]:

$$l_i^- \le \frac{g_i(x_1) - g_i(x_2)}{x_1 - x_2} \le l_i^+ \tag{3}$$

where l_i^- and l_i^+ are known constants.

Remark 1: It is difficult to obtain the accurate weight of the connection among the neurons, the norm bounded model [35] and the interval model [36] have been used to deal with the uncertain weight matrix. This paper applies the stochastic variable $b_{ij,\theta(k)}(k)$ to describe the uncertain weight. According to (2), the merit of this model is that it can reflect the interaction among the weight matrices. In addition, the stochastic properties of the weight matrices are periodical, which is consistent with the mode of the NNs.

The periodic NNs (1) can be further expressed as

$$\mathbf{x}(k+1) = A_{\theta(k)}\mathbf{x}(k) + \Big(B_{\theta(k)} + \check{B}_{\theta(k)}(k)\Big)\mathbf{g}(\mathbf{x}(k)) + E_{\theta(k)}\mathbf{w}(k) \tag{4}$$

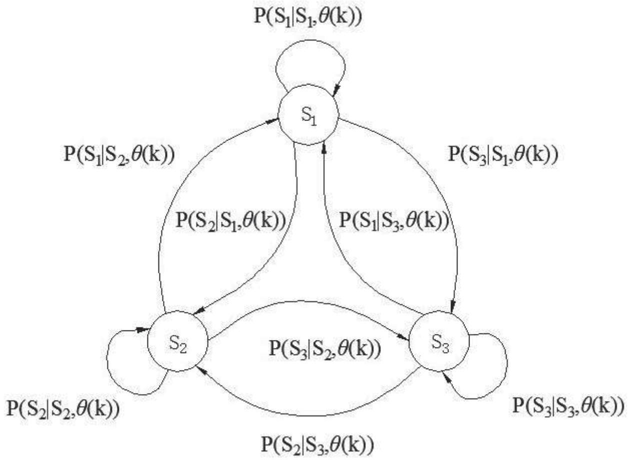


Fig. 1. Mode-dependent Markov chain with three states.

where

$$\begin{aligned}
\mathbf{g}(\mathbf{x}(k)) &= \left[g_1(x_1(k))\ g_2(x_2(k))\dots\ g_n(x_n(k))\right]^{\mathrm{T}}\\
\mathbf{x}(k) &= [x_1(k)\ x_2(k)\dots x_n(k)]^{\mathrm{T}}\\
\mathbf{w}(k) &= [w_1(k)\ w_2(k)\dots w_n(k)]^{\mathrm{T}}\\
A_{\theta(k)} &= \mathrm{diag}\left\{a_{1,\theta(k)}, a_{2,\theta(k)}, \dots, a_{n,\theta(k)}\right\}\\
B_{\theta(k)} &= \left[\bar{b}_{ij,\theta(k)}\right]_{n\times n},\ \check{B}_{\theta(k)}(k) = \left[\check{b}_{ij,\theta(k)}(k)\right]_{n\times n}\\
E_{\theta(k)} &= \mathrm{diag}\left\{e_{1,\theta(k)}, e_{2,\theta(k)}, \dots,\ e_{n,\theta(k)}\right\}.
\end{aligned}$$

The measurement and the signal to be estimated are denoted as follows, respectively:

$$\begin{aligned}
\mathbf{y}(k) &= C_{\theta(k)}\mathbf{x}(k) + D_{\theta(k)}\mathbf{w}(k)\\
\mathbf{z}(k) &= S_{\theta(k)}\mathbf{x}(k)
\end{aligned}\tag{5}$$

where $C_{\theta(k)} \in \mathbb{R}^{p\times n}$, $D_{\theta(k)} \in \mathbb{R}^{p\times n}$, and $S_{\theta(k)} \in \mathbb{R}^{q\times n}$ are m-periodic known matrices.

Measurements obtained via the sensors are always transmitted through the wireless communication channels. However, for the wireless channels, they are easily influenced by the transmission energy and the environment. Therefore, the channel states are time varying [38]. In this paper, Markov chain $\delta(k) \in \mathcal{N}_c \triangleq \{1, 2, \dots, s\}$, depending on the system mode $\theta(k)$, is used to describe the variation of the channel states, and the transition probability matrices $\Pi^{\iota} \triangleq \{\pi^{\iota}_{\varpi\nu}\}$ are assumed to be m-periodic and given by

$$\pi^{\iota}_{\varpi\nu} = \mathbb{P}\{\delta(k+1) = \nu | \delta(k) = \varpi, \theta(k) = \iota\}\tag{6}$$

where $0 \le \pi^{\iota}_{\varpi\nu} \le 1$, $\forall\varpi, \nu \in \mathcal{N}_c$, and $\iota \in \mathcal{N}_m$, and $\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu} = 1$, $\forall\varpi \in \mathcal{N}_c$, and $\iota \in \mathcal{N}_m$.

Remark 2: Note that the transition probability (6) of Markov chain depends not only on the state $\delta(k)$, but also on the mode of the periodic NNs. An example of three states of such Markov chain is shown in Fig. 1. The advantage of using this Markov chain is that the channel states are related to the mode of the NNs. For example, if the mode of the NNs $\iota \in \mathcal{N}_m$ is important, the channel state can be enhanced by increasing the transmission energy to improve the performance of the NNs [18], [39].

For wireless communication channel, packet dropouts are unavoidable [40]. This paper uses the channel-state-dependent Bernoulli process $\xi_{\delta(k)}(k)$ to describe the packet dropouts, that is, the measurement $\tilde{\mathbf{y}}(k)$ received by the estimator is

$$\tilde{\mathbf{y}}(k) = \xi_{\delta(k)}(k)\mathbf{y}(k)\tag{7}$$

where Bernoulli process $\xi_{\delta(k)}(k)$ satisfies the following stochastic properties:

$$\begin{aligned}
\mathbb{E}\{\xi_{\delta(k)}(k)\} &= \bar{\xi}_{\delta(k)}\\
\mathbb{E}\left\{\left(\xi_{\delta(k)}(k) - \bar{\xi}_{\delta(k)}\right)^2\right\} &= \xi^*_{\delta(k)}.
\end{aligned}\tag{8}$$

Remark 3: Packet dropouts have been studied by a number of researchers in the past two decades. Bernoulli process and Markov chain are the main stochastic models to describe packet dropouts of NCSs. This paper studies the case where the packet dropout rate depends on the channel states, which is more general than the existing ones [26].

B. State Estimator

For the NNs (4) and (5), the state estimator is synthesized as

$$\begin{aligned}
\hat{\mathbf{x}}(k+1) &= A_{\theta(k)}\hat{\mathbf{x}}(k) + B_{\theta(k)}\mathbf{g}\left(\hat{\mathbf{x}}(k)\right)\\
&\quad + \xi_{\delta(k)}(k)K_{\theta(k),\delta(k)}\left(\mathbf{y}(k) - C_{\theta(k)}\hat{\mathbf{x}}(k)\right)\\
\hat{\mathbf{z}}(k) &= S_{\theta(k)}\hat{\mathbf{x}}(k)
\end{aligned}\tag{9}$$

where $\hat{\mathbf{x}}(k) \in \mathbb{R}^n$, $\hat{\mathbf{z}}(k) \in \mathbb{R}^q$, and $K_{\theta(k),\delta(k)}$ is the state estimator gain to be designed.

Let the estimation error be $\mathbf{e}(k) \triangleq \mathbf{x}(k) - \hat{\mathbf{x}}(k)$, and $\tilde{\mathbf{z}}(k) \triangleq \mathbf{z}(k) - \hat{\mathbf{z}}(k)$. Then the state error dynamic is obtained as

$$\begin{aligned}
\mathbf{e}(k+1) &= A_{\theta(k)}\mathbf{e}(k) + B_{\theta(k)}\tilde{\mathbf{g}}\left(\mathbf{x}(k), \hat{\mathbf{x}}(k)\right)\\
&\quad + \check{B}_{\theta(k)}\mathbf{g}(\mathbf{x}(k)) + E_{\theta(k)}\mathbf{w}(k)\\
&\quad - \xi_{\delta(k)}(k)K_{\theta(k),\delta(k)}\left(C_{\theta(k)}\mathbf{e}(k) + D_{\theta(k)}\mathbf{w}(k)\right)\\
\tilde{\mathbf{z}}(k) &= S_{\theta(k)}\mathbf{e}(k)
\end{aligned}\tag{10}$$

where $\tilde{\mathbf{g}}(\mathbf{x}(k), \hat{\mathbf{x}}(k)) \triangleq \mathbf{g}(\mathbf{x}(k)) - \mathbf{g}(\hat{\mathbf{x}}(k))$.

Define $\tilde{\mathbf{x}}(k) \triangleq [\mathbf{x}^{\mathrm{T}}(k)\ \ \mathbf{e}^{\mathrm{T}}(k)]^{\mathrm{T}}$, and $\Psi(k) \triangleq [\mathbf{g}^{\mathrm{T}}(\mathbf{x}(k))\ \tilde{\mathbf{g}}^{\mathrm{T}}(\mathbf{x}(k), \hat{\mathbf{x}}(k))]^{\mathrm{T}}$. Then by combining (4) and (10), au augmented system can be expressed as

$$\begin{aligned}
\tilde{\mathbf{x}}(k+1) &= \tilde{A}_{\theta(k),\delta(k)}\tilde{\mathbf{x}}(k) + \tilde{\xi}_{\delta(k)}(k)\bar{A}_{\theta(k),\delta(k)}\tilde{\mathbf{x}}(k)\\
&\quad + \left(\tilde{B}_{\theta(k)} + \mathscr{B}_{\theta(k)}(k)\right)\Psi(k) + \tilde{E}_{\theta(k)}\mathbf{w}(k)\\
&\quad + \tilde{\xi}_{\delta(k)}(k)\bar{E}_{\theta(k),\delta(k)}\mathbf{w}(k)\\
\tilde{\mathbf{z}}(k) &= \bar{S}_{\theta(k)}\tilde{\mathbf{x}}(k)
\end{aligned}\tag{11}$$

where

$$\begin{aligned}
\tilde{A}_{\theta(k),\delta(k)} &= \begin{bmatrix} A_{\theta(k)} & 0\\ 0 & A_{\theta(k)} - \bar{\xi}_{\delta(k)}K_{\theta(k),\delta(k)}C_{\theta(k)}\end{bmatrix}\\
\bar{A}_{\theta(k),\delta(k)} &= \begin{bmatrix} 0 & 0\\ 0 & -K_{\theta(k),\delta(k)}C_{\theta(k)}\end{bmatrix}\\
\mathscr{B}_{\theta(k)}(k) &= \begin{bmatrix} \check{B}_{\theta(k)}(k) & 0\\ \check{B}_{\theta(k)}(k) & 0\end{bmatrix},\ \tilde{B}_{\theta(k)} = \begin{bmatrix} B_{\theta(k)} & 0\\ 0 & B_{\theta(k)}\end{bmatrix}\\
\tilde{E}_{\theta(k)} &= \begin{bmatrix} E_{\theta(k)}\\ E_{\theta(k)} - \bar{\xi}_{\delta(k)}K_{\theta(k),\delta(k)}D_{\theta(k)}\end{bmatrix}\\
\bar{E}_{\theta(k),\delta(k)} &= \begin{bmatrix} 0\\ -K_{\theta(k),\delta(k)}D_{\theta(k)}\end{bmatrix}\\
\bar{S}_{\theta(k)} &= \left[0\ S_{\theta(k)}\right],\ \tilde{\xi}_{\delta(k)}(k) = \xi_{\delta(k)}(k) - \bar{\xi}_{\delta(k)}.
\end{aligned}$$

The following definitions are recalled for the main results.

Definition 1 [41]: The augmented system (11) is stochastically stable with $\mathbf{w}(k) = 0$, if the following inequality holds $\forall \tilde{\mathbf{x}}(0)$, and $\delta(0) \in \mathcal{N}_c$:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\left\|\tilde{\mathbf{x}}(k)|_{\tilde{\mathbf{x}}(0),\delta(0)}\right\|^2\right\} < +\infty.$$

Definition 2 [42]: Given a scalar $\gamma > 0$, the augmented system (11) is said to be stochastically stable with H_∞ performance γ, if the system with $\mathbf{w}(k) = 0$ is stochastically stable, and for all nonzero input $\mathbf{w}(k) \in l_2[0, +\infty)$, the following requirement holds under zero initial condition:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\left\|\tilde{\mathbf{z}}(k)\big|_{\delta(0)}\right\|^2\right\} \le \gamma^2 \sum_{k=0}^{\infty}\|\mathbf{w}(k)\|^2.$$

The aim of this paper is to construct a state estimator of (9), which depends on the system mode and the channel state, such that the following two conditions are satisfied simultaneously.

1) The augmented system (11) is stochastically stable with $\mathbf{w}(k) = 0$.
2) Under zero initial condition, for all nonzero $\mathbf{w}(k) \in l_2[0, +\infty)$, the augmented system (11) is stochastically stable with H_∞ performance γ.

III. Main Results

The sufficient condition of the stochastic stability and the H_∞ performance for the augmented system (11) has been obtained, which is given as follows.

Theorem 1: Given a scalar $\gamma > 0$, the augmented system (11) is stochastically stable and has an H_∞ performance index γ, if matrices $P_{\iota,\varpi} > 0$, $M > 0$, and $R > 0$ can be found such that the inequalities in (12) hold $\forall \iota \in \mathcal{N}_m, \varpi \in \mathcal{N}_c$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \tilde{A}^{\mathrm{T}}_{\iota,\varpi} & \Phi_{15} & \bar{S}^{\mathrm{T}}_{\iota} \\ * & \Phi_{22} & 0 & \tilde{B}^{\mathrm{T}}_{\iota} & 0 & 0 \\ * & * & -\gamma^2 I & \tilde{E}^{\mathrm{T}}_{\iota} & \Phi_{35} & 0 \\ * & * & * & -\Phi_{44} & 0 & 0 \\ * & * & * & * & -\Phi_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{12}$$

where

$$L_1 = \mathrm{diag}\{l_1^-, l_2^-, \dots, l_n^-\}, L_2 = \mathrm{diag}\{l_1^+, l_2^+, \dots, l_n^+\}$$
$$M = \mathrm{diag}\{m_1, m_2, \dots, m_n\}, R = \mathrm{diag}\{r_1, r_2, \dots, r_n\}$$
$$\Phi_{11} = -2\begin{bmatrix} L_2^{\mathrm{T}} M L_1 & 0 \\ 0 & L_2^{\mathrm{T}} R L_1 \end{bmatrix} - P_{\iota,\varpi}$$
$$\Phi_{12} = \begin{bmatrix} L_2^{\mathrm{T}} M + L_1^{\mathrm{T}} M^{\mathrm{T}} & 0 \\ 0 & L_2^{\mathrm{T}} R + L_1^{\mathrm{T}} R^{\mathrm{T}} \end{bmatrix}$$
$$\Phi_{15} = \sqrt{\xi^*_{\varpi}}\bar{A}^{\mathrm{T}}_{\iota,\varpi}, \quad \Phi_{35} = \sqrt{\xi^*_{\varpi}}\bar{E}^{\mathrm{T}}_{\iota,\varpi}$$
$$\Phi_{22} = \begin{bmatrix} -2M & 0 \\ 0 & -2R \end{bmatrix} + \mathbb{E}\left\{\mathscr{B}^{\mathrm{T}}_{\iota}(k) P_{\varsigma,\nu}\mathscr{B}_{\iota}(k)\right\}$$
$$\Phi_{44} = \Phi_{55} = \left(\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\right)^{-1}.$$

Proof: The stochastic stability of the augmented system (11) without noises (i.e., $\mathbf{w}(k) = 0$) is considered first. The following Lyapunov function is constructed:

$$\mathcal{V}\big(\tilde{\mathbf{x}}(k), \theta(k), \delta(k)\big) = \tilde{\mathbf{x}}^{\mathrm{T}}(k) P_{\theta(k),\delta(k)}\tilde{\mathbf{x}}(k) \tag{13}$$

where $P_{\theta(k),\delta(k)} > 0$. Let $\theta(k) = \iota$, $\theta(k+1) = \varsigma$, $\delta(k) = \varpi$, and $\delta(k+1) = \nu$, then the expectation of $\triangle\mathcal{V}(\tilde{\mathbf{x}}(k), \theta(k), \delta(k))$ is defined as

$$\begin{aligned}\mathbb{E}&\big\{\triangle\mathcal{V}\big(\tilde{\mathbf{x}}(k), \iota, \varpi\big)\big\} \\ &\triangleq \mathbb{E}\big\{\mathcal{V}\big(\tilde{\mathbf{x}}(k+1), \varsigma, \nu\big)|_{\tilde{\mathbf{x}}(k),\iota,\varpi}\big\} - \mathcal{V}\big(\tilde{\mathbf{x}}(k), \iota, \varpi\big) \\ &= \mathbb{E}\big\{\tilde{\mathbf{x}}^{\mathrm{T}}(k+1)P_{\varsigma,\nu}\tilde{\mathbf{x}}(k+1)\big\} - \tilde{\mathbf{x}}^{\mathrm{T}}(k)P_{\iota,\varpi}\tilde{\mathbf{x}}(k).\end{aligned} \tag{14}$$

Then, substituting the augmented system (11) without noise term into (14) yields

$$\begin{aligned}\mathbb{E}\big\{\triangle\mathcal{V}\big(\tilde{\mathbf{x}}(k), \iota, \varpi\big)\big\} &= \tilde{\mathbf{x}}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{A}_{\iota,\varpi}\tilde{\mathbf{x}}(k) \\ &\quad + \xi^*_{\varpi}\tilde{\mathbf{x}}^{\mathrm{T}}(k)\bar{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\bar{A}_{\iota,\varpi}\tilde{\mathbf{x}}(k) \\ &\quad + 2\tilde{\mathbf{x}}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}\Psi(k) \\ &\quad + \Psi^{\mathrm{T}}(k)\tilde{B}^{\mathrm{T}}_{\iota}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}\Psi(k) \\ &\quad + \Psi^{\mathrm{T}}(k)\mathbb{E}\big\{\mathscr{B}^{\mathrm{T}}_{\iota}(k)P_{\varsigma,\nu}\mathscr{B}_{\iota}(k)\big\}\Psi(k) \\ &\quad - \tilde{\mathbf{x}}^{\mathrm{T}}(k)P_{\iota,\varpi}\tilde{\mathbf{x}}(k).\end{aligned} \tag{15}$$

According to (3), the nonlinear neuron activation function $g_i(\cdot)$ satisfies

$$\begin{aligned}\frac{g_i(x_i(k)) - l_i^- x_i(k)}{x_i(k)} &\ge 0 \\ \frac{g_i(x_i(k)) - l_i^+ x_i(k)}{x_i(k)} &\le 0.\end{aligned} \tag{16}$$

For any positive scalars $m_h > 0$ and $r_h > 0$, $h = 1, 2, \dots, n$, we have the following inequalities:

$$-2\sum_{h=1}^{n} m_h(g_h(x_h(k)) - l_h^+ x_h(k))(g_h(x_h(k)) - l_h^- x_h(k)) \ge 0 \tag{17}$$

and

$$\begin{aligned}-2\sum_{h=1}^{n} & r_h(\tilde{g}_h(x_h(k), \hat{x}_h(k)) - l_h^+ e_h(k)) \\ &\times \big(\tilde{g}_h\big(x_h(k), \hat{x}_h(k)\big) - l_h^- e_h(k)\big) \ge 0.\end{aligned} \tag{18}$$

The conditions (17) and (18) imply that

$$-2(\mathbf{g}(\mathbf{x}(k)) - L_2\mathbf{x}(k))^{\mathrm{T}} M(\mathbf{g}(\mathbf{x}(k)) - L_1\mathbf{x}(k)) \ge 0 \tag{19}$$

and

$$\begin{aligned}-2\big(\tilde{\mathbf{g}}&\big(\mathbf{x}(k), \hat{\mathbf{x}}(k)\big) - L_2\mathbf{e}(k)\big)^{\mathrm{T}} R \\ &\times \big(\tilde{\mathbf{g}}\big(\mathbf{x}(k), \hat{\mathbf{x}}(k)\big) - L_1\mathbf{e}(k)\big) \ge 0.\end{aligned} \tag{20}$$

Substituting the conditions of (19) and (20) into the difference of the Lyapunov function (15) in the mean senses, one has

$$\mathbb{E}\big\{\triangle\mathcal{V}\big(\tilde{\mathbf{x}}(k),\iota,\varpi\big)\big\} \le \eta^{\mathrm{T}}(k)\Omega\eta(k)$$

with

$$\eta^{\mathrm{T}}(k) = \begin{bmatrix}\tilde{\mathbf{x}}^{T}(k) & \boldsymbol{\Psi}^{T}(k)\end{bmatrix},\ \Omega = \begin{bmatrix}\Omega_{11} & \Omega_{12}\\ * & \Omega_{22}\end{bmatrix}$$

$$\begin{aligned}\Omega_{11} &= \tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{A}_{\iota,\varpi}\\ &\quad+\xi^{*}_{\varpi}\bar{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\bar{A}_{\iota,\varpi}\\ &\quad-2\begin{bmatrix}L_2^{\mathrm{T}}ML_1 & 0\\ 0 & L_2^{\mathrm{T}}RL_1\end{bmatrix}-P_{\iota,\varpi}\\ \Omega_{12} &= \tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}\\ &\quad+\begin{bmatrix}L_2^{\mathrm{T}}M+L_1^{\mathrm{T}}M^{\mathrm{T}} & 0\\ 0 & L_2^{\mathrm{T}}R+L_1^{\mathrm{T}}R^{\mathrm{T}}\end{bmatrix}\\ \Omega_{22} &= \tilde{B}^{\mathrm{T}}_{\iota}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}+\mathbb{E}\big\{\mathscr{B}^{\mathrm{T}}_{\iota}(k)P_{\varsigma,\nu}\mathscr{B}_{\iota}(k)\big\}\\ &\quad-2\begin{bmatrix}M & 0\\ 0 & R\end{bmatrix}.\end{aligned}$$

By using the Schur complement lemma to (12), the inequality $\mathbb{E}\{\triangle\mathcal{V}(\tilde{\mathbf{x}}(k),\theta(k),\varpi)\} < 0$ holds. Thus, we can derive the augmented system (11) is stochastically stable on the basis of the method proposed in [43].

Next, we begin to study the H_∞ performance of the augmented system (11) with disturbance $\mathbf{w}(k)$. We have

$$\begin{aligned}&\mathbb{E}\big\{\triangle\mathcal{V}\big(\tilde{\mathbf{x}}(k),\iota,\varpi\big)\big\}\\ &\quad=\mathbb{E}\big\{\tilde{\mathbf{x}}^{\mathrm{T}}(k+1)P_{\varsigma,\nu}\tilde{\mathbf{x}}(k+1)\big\}-\tilde{\mathbf{x}}^{\mathrm{T}}(k)P_{\iota,\varpi}\tilde{\mathbf{x}}(k)\\ &\quad=\tilde{\mathbf{x}}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{A}_{\iota,\varpi}\tilde{\mathbf{x}}(k)\\ &\qquad+\xi^{*}_{\varpi}\tilde{\mathbf{x}}^{\mathrm{T}}(k)\bar{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\bar{A}_{\iota,\varpi}\tilde{\mathbf{x}}(k)\\ &\qquad+2\tilde{\mathbf{x}}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{E}_{\iota}\mathbf{w}(k)\\ &\qquad+2\xi^{*}_{\varpi}\tilde{\mathbf{x}}^{\mathrm{T}}(k)\bar{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\bar{E}_{\iota,\varpi}\mathbf{w}(k)\\ &\qquad+2\tilde{\mathbf{x}}^{\mathrm{T}}(k)\tilde{A}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}\Psi(k)\\ &\qquad+2\mathbf{w}^{\mathrm{T}}(k)\tilde{E}^{\mathrm{T}}_{\iota}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}\Psi(k)\\ &\qquad+\Psi^{\mathrm{T}}(k)\tilde{B}^{\mathrm{T}}_{\iota}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{B}_{\iota}\Psi(k)\\ &\qquad+\Psi^{\mathrm{T}}(k)\mathbb{E}\big\{\mathscr{B}^{\mathrm{T}}_{\iota}(k)P_{\varsigma,\nu}\mathscr{B}_{\iota}(k)\big\}\Psi(k)\\ &\qquad+\xi^{*}_{\varpi}\mathbf{w}^{\mathrm{T}}(k)\bar{E}^{\mathrm{T}}_{\iota,\varpi}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\bar{E}_{\iota,\varpi}\mathbf{w}(k)\\ &\qquad+\mathbf{w}^{\mathrm{T}}(k)\tilde{E}^{\mathrm{T}}_{\iota}\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\tilde{E}_{\iota}\mathbf{w}(k)-\tilde{\mathbf{x}}^{\mathrm{T}}(k)P_{\iota,\varpi}\tilde{\mathbf{x}}(k).\end{aligned}\tag{21}$$

Define the following index $J(\infty)$:

$$\begin{aligned}J(\infty) &= \sum_{k=0}^{\infty}\mathbb{E}\Big\{\tilde{\mathbf{z}}^{\mathrm{T}}(k)\tilde{\mathbf{z}}(k)-\gamma^2\mathbf{w}^{\mathrm{T}}(k)\mathbf{w}(k)\Big\}\\ &= \sum_{k=0}^{\infty}\mathbb{E}\Big\{\tilde{\mathbf{z}}^{\mathrm{T}}(k)\tilde{\mathbf{z}}(k)-\gamma^2\mathbf{w}^{\mathrm{T}}(k)\mathbf{w}(k)+\triangle\mathcal{V}\big(\tilde{\mathbf{x}}(k),\iota,\varpi\big)\Big\}\\ &\quad-\mathbb{E}\big\{\mathcal{V}\big(\tilde{\mathbf{x}}(\infty),\theta(\infty),\delta(\infty)\big)\big\}+\mathcal{V}\big(\tilde{\mathbf{x}}(0),\theta(0),\delta(0)\big).\end{aligned}\tag{22}$$

Considering the fact $\mathbb{E}\{V(\tilde{\mathbf{x}}(\infty),\theta(\infty),\delta(\infty))\} \ge 0$, the zero initial condition, and the inequalities of the nonlinearities (19) and (20), the following condition holds from (12) and (22):

$$J(\infty) \le \sum_{k=0}^{\infty}\mathbb{E}\big\{\eta^{\mathrm{T}}(k)\Phi\eta(k)\big\} < 0 \tag{23}$$

where

$$\eta^{\mathrm{T}}(k) \triangleq \begin{bmatrix}\tilde{\mathbf{x}}^{\mathrm{T}}(k) & \Psi^{\mathrm{T}}(k) & \mathbf{w}^{\mathrm{T}}(k)\end{bmatrix}.$$

Thus we obtain that the augmented system (11) has H_∞ performance γ. ■

In order to facilitate the estimator gain design, a new theorem is obtained.

Theorem 2: Given a scalar $\gamma > 0$, the augmented system (11) is stochastically stable and has an H_∞ performance index γ, if there exist matrices $P_{\iota,\varpi} > 0$, $M > 0$, $R > 0$, and $W_{\iota,\varpi}$ such that the inequalities in (24) hold $\forall\iota \in \mathcal{N}_m$, $\varpi \in \mathcal{N}_c$

$$\begin{bmatrix}\Phi_{11} & \Phi_{12} & 0 & \Gamma_{14} & \Gamma_{15} & \bar{S}^{\mathrm{T}}_{\iota}\\ * & \Phi_{22} & 0 & \Gamma_{24} & 0 & 0\\ * & * & -\gamma^2 I & \Gamma_{34} & \Gamma_{35} & 0\\ * & * & * & \Gamma_{44} & 0 & 0\\ * & * & * & * & \Gamma_{55} & 0\\ * & * & * & * & * & -I\end{bmatrix} < 0 \tag{24}$$

where

$$\begin{aligned}&\Gamma_{14} = \tilde{A}^{\mathrm{T}}_{\iota,\varpi}W^{\mathrm{T}}_{\iota,\varpi},\ \Gamma_{15} = \sqrt{\xi^{*}_{\varpi}}\bar{A}^{\mathrm{T}}_{\iota,\varpi}W^{\mathrm{T}}_{\iota,\varpi}\\ &\Gamma_{24} = \tilde{B}^{\mathrm{T}}_{\iota}W^{\mathrm{T}}_{\iota,\varpi},\ \Gamma_{34} = \tilde{E}^{\mathrm{T}}_{\iota}W^{\mathrm{T}}_{\iota,\varpi}\\ &\Gamma_{44} = \Gamma_{55} = \sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}-W_{\iota,\varpi}-W^{\mathrm{T}}_{\iota,\varpi}\\ &\Gamma_{35} = \sqrt{\xi^{*}_{\varpi}}\bar{E}^{\mathrm{T}}_{\iota,\varpi}W^{\mathrm{T}}_{\iota,\varpi}.\end{aligned}$$

Proof: For any matrices $W_{\iota,\varpi}$, $\forall\iota \in \mathcal{N}_m$, and $\forall\varpi \in \mathcal{N}_c$, the following inequality holds:

$$\left(\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}-W_{\iota,\varpi}\right)\left(\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}\right)^{-1}\times\left(\sum_{\nu=1}^{s}\pi^{\iota}_{\varpi\nu}P_{\varsigma,\nu}-W_{\iota,\varpi}\right)^{\mathrm{T}} \ge 0$$

which implies

$$-W_{\iota,\varpi}\left(\sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}\right)^{-1}W_{\iota,\varpi}^{\mathrm{T}} \le \sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu} - W_{\iota,\varpi} - W_{\iota,\varpi}^{\mathrm{T}}. \quad (25)$$

Hence, based on (25), we derive the following inequalities hold $\forall\iota \in \mathcal{N}_m$ and $\varpi \in \mathcal{N}_c$ from (24):

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \Gamma_{14} & \Gamma_{15} & \bar{S}_\iota^{\mathrm{T}} \\ * & \Phi_{22} & 0 & \Gamma_{24} & 0 & 0 \\ * & * & -\gamma^2 I & \Gamma_{34} & \Gamma_{35} & 0 \\ * & * & * & \Lambda_{44} & 0 & 0 \\ * & * & * & * & \Lambda_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (26)$$

where

$$\Lambda_{44} = \Lambda_{55} = -W_{\iota,\varpi}\left(\sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}\right)^{-1}W_{\iota,\varpi}^{\mathrm{T}}.$$

Performing a congruence transformation using $\mathrm{diag}\{I, I, I, W_{\iota,\varpi}^{-1}, W_{\iota,\varpi}^{-1}, I\}$ yields (12), that is, the stochastic stability and the H_∞ performance of the augmented system (11) are ensured. ■

IV. Estimator Design

What we have to point out is that the inequalities (24) in Theorem 2 are not LMIs. Therefore, the result of Theorem 2 is going to be transformed by using linearization technique. The parameters of the state estimator in (9) can be designed in the following theorem.

Theorem 3: Given a scalar $\gamma > 0$, the augmented system (11) is stochastically stable and has an H_∞ performance index γ, if there exist matrices

$$\begin{bmatrix} P_{\iota,\varpi}^1 & P_{\iota,\varpi}^2 \\ * & P_{\iota,\varpi}^3 \end{bmatrix} > 0, \begin{bmatrix} W_{\iota,\varpi}^1 & W_{\iota,\varpi}^2 \\ W_{\iota,\varpi}^3 & W_{\iota,\varpi}^2 \end{bmatrix},\ M > 0, \quad R > 0$$

and $\mathcal{K}_{\iota,\varpi}$ such that the inequalities in (27) hold $\forall\iota \in \mathcal{N}_m$, $\varpi \in \mathcal{N}_c$

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & 0 & \Upsilon_{14} & \sqrt{\xi_\varpi^*}\Upsilon_{15} & \bar{S}_\iota^{\mathrm{T}} \\ * & \Upsilon_{22} & 0 & \Upsilon_{24} & 0 & 0 \\ * & * & -\gamma^2 I & \Upsilon_{34} & \sqrt{\xi_\varpi^*}\Upsilon_{35} & 0 \\ * & * & * & \Upsilon_{44} & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (27)$$

where

$$\Upsilon_{11} = \begin{bmatrix} -P_{\iota,\varpi}^1 - 2L_2^{\mathrm{T}}ML_1 & -P_{\iota,\varpi}^2 \\ * & -P_{\iota,\varpi}^3 - 2L_2^{\mathrm{T}}RL_1 \end{bmatrix}$$

$$\Upsilon_{12} = \begin{bmatrix} L_2^{\mathrm{T}}M + L_1^{\mathrm{T}}M^{\mathrm{T}} & 0 \\ 0 & L_2^{\mathrm{T}}R + L_1^{\mathrm{T}}R^{\mathrm{T}} \end{bmatrix}$$

$$\Upsilon_{14} = \begin{bmatrix} W_{\iota,\varpi}^1 A_\iota & W_{\iota,\varpi}^2 A_\iota - \bar{\xi}_\varpi \mathcal{K}_{\iota,\varpi} C_\iota \\ W_{\iota,\varpi}^3 A_\iota & W_{\iota,\varpi}^2 A_\iota - \bar{\xi}_\varpi \mathcal{K}_{\iota,\varpi} C_\iota \end{bmatrix}^{\mathrm{T}}$$

$$\Upsilon_{15} = \begin{bmatrix} 0 & -\mathcal{K}_{\iota,\varpi} C_\iota \\ 0 & -\mathcal{K}_{\iota,\varpi} C_\iota \end{bmatrix}^{\mathrm{T}},\ \Upsilon_{35} = \begin{bmatrix} -\mathcal{K}_{\iota,\varpi} D_\iota \\ -\mathcal{K}_{\iota,\varpi} D_\iota \end{bmatrix}^{\mathrm{T}}$$

$$\Upsilon_{22} = \begin{bmatrix} -2M & 0 \\ 0 & -2R \end{bmatrix} + \begin{bmatrix} \Omega_{r\nu} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned}\Omega_{r\nu} = &\sum_{\nu=1}^{s}\sum_{j=1}^{n}\sum_{i=1}^{n}\pi_{\varpi\nu}^{\iota}\varrho_{irj\sigma,\iota}\left(P_{\varsigma,\nu}^1\right)_{ij} \\ &+ 2\sum_{\nu=1}^{s}\sum_{j=1}^{n}\sum_{i=1}^{n}\pi_{\varpi\nu}^{\iota}\varrho_{irj\sigma,\iota}\left(P_{\varsigma,\nu}^2\right)_{ij} \\ &+ \sum_{\nu=1}^{s}\sum_{j=1}^{n}\sum_{i=1}^{n}\pi_{\varpi\nu}^{\iota}\varrho_{irj\sigma,\iota}\left(P_{\varsigma,\nu}^3\right)_{ij}\end{aligned}$$

$$\Upsilon_{24} = \begin{bmatrix} W_{\iota,\varpi}^1 B_\iota & W_{\iota,\varpi}^2 B_\iota \\ W_{\iota,\varpi}^3 B_\iota & W_{\iota,\varpi}^2 B_\iota \end{bmatrix}^{\mathrm{T}}$$

$$\Upsilon_{34} = \begin{bmatrix} \left(W_{\iota,\varpi}^1 + W_{\iota,\varpi}^2\right) E_\iota - \bar{\xi}_\varpi^* \mathcal{K}_{\iota,\varpi} D_\iota \\ \left(W_{\iota,\varpi}^2 + W_{\iota,\varpi}^3\right) E_\iota - \bar{\xi}_\varpi^* \mathcal{K}_{\iota,\varpi} D_\iota \end{bmatrix}^{\mathrm{T}}$$

$$\Upsilon_{44} = \Upsilon_{55} = \begin{bmatrix} \sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}^1 - W_{\iota,\varpi}^1 - W_{\iota,\varpi}^{1\mathrm{T}} & \sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}^2 - W_{\iota,\varpi}^2 - W_{\iota,\varpi}^{3\mathrm{T}} \\ * & \sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}^3 - W_{\iota,\varpi}^2 - W_{\iota,\varpi}^{2\mathrm{T}} \end{bmatrix}.$$

Then, the parameters of the state estimator in (9) are given as

$$K_{\iota,\varpi} = \left(W_{\iota,\varpi}^2\right)^{-1}\mathcal{K}_{\iota,\varpi}. \quad (28)$$

Proof: Define the partition matrices of $P_{\iota,\varpi}$ and $W_{\iota,\varpi}$ in (24) as

$$P_{\iota,\varpi} \triangleq \begin{bmatrix} P_{\iota,\varpi}^1 & P_{\iota,\varpi}^2 \\ * & P_{\iota,\varpi}^3 \end{bmatrix},\ W_{\iota,\varpi} \triangleq \begin{bmatrix} W_{\iota,\varpi}^1 & W_{\iota,\varpi}^2 \\ W_{\iota,\varpi}^3 & W_{\iota,\varpi}^2 \end{bmatrix}. \quad (29)$$

Then we have

$$\mathbb{E}\left\{\mathscr{B}_\iota^{\mathrm{T}}(k)\sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}\mathscr{B}_\iota(k)\right\} = \begin{bmatrix} \digamma & 0 \\ 0 & 0 \end{bmatrix}$$

where

$$\begin{aligned}\digamma = &\mathbb{E}\left\{\check{B}_\iota^{\mathrm{T}}(k)\sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}^1\check{B}_\iota(k)\right\} \\ &+ 2\mathbb{E}\left\{\check{B}_\iota^{\mathrm{T}}(k)\sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}^2\check{B}_\iota(k)\right\} \\ &+ \mathbb{E}\left\{\check{B}_\iota^{\mathrm{T}}(k)\sum_{\nu=1}^{s}\pi_{\varpi\nu}^{\iota}P_{\varsigma,\nu}^3\check{B}_\iota(k)\right\}.\end{aligned}$$

Since the matrix multiplication is not commutable, every entry of the matrix has to be computed. First, the (r, σ)' entry of the matrix $\check{B}_\iota^{\mathrm{T}}(k)P_{\varsigma,\nu}^1$ is

$$\left(\check{B}_\iota^{\mathrm{T}}(k)P_{\varsigma,\nu}^1\right)_{r\sigma} = \sum_{m=1}^{n}\left(\check{B}_\iota(k)\right)_{mr}\left(P_{\varsigma,\nu}^1\right)_{m\sigma}.$$

Furthermore, we obtain that

$$\begin{aligned}&\left(\check{B}_\iota^{\mathrm{T}}(k)P_{\varsigma,\nu}^1\check{B}_\iota(k)\right)_{r\sigma} \\ &= \sum_{m=1}^{n}\left(\check{B}_\iota(k)\right)_{mr}\left(P_{\varsigma,\nu}^1\right)_{m1}\left(\check{B}_\iota(k)\right)_{1\sigma} \\ &\quad + \ldots + \sum_{m=1}^{n}\left(\check{B}_\iota(k)\right)_{mr}\left(P_{\varsigma,\nu}^1\right)_{mn}\left(\check{B}_\iota(k)\right)_{n\sigma}. \quad (30)\end{aligned}$$

Since $(\breve{B}_\iota(k))_{ij}$, $(P^1_{\varsigma,\nu})_{\alpha\beta}$ $(i, j, \alpha, \beta = 1, 2, \ldots, n)$ are scalars, they are multiplication commutable. According to the properties in (2), we have

$$\mathbb{C}\left\{\left(\breve{B}_\iota(k)\right)_{ij}, \left(\breve{B}_\iota(k)\right)_{\alpha\beta}\right\} = \varrho_{ij\alpha\beta,\iota}. \tag{31}$$

It follows from (30) and (31) that

$$\mathbb{E}\left\{\breve{B}^{\mathrm{T}}_\iota(k)P^1_{\varsigma,\iota}\breve{B}_\iota(k)\right\}_{r\sigma} = \sum_{i=1}^{n}\varrho_{ir1\sigma,\iota}\left(P^1_{\varsigma,\nu}\right)_{i1} + \cdots + \sum_{i=1}^{n}\varrho_{irn\sigma,\iota}\left(P^1_{\varsigma,\nu}\right)_{in}.$$

Then, we obtain that

$$\mathbb{E}\left\{\breve{B}^{\mathrm{T}}_\iota(k)P^1_{\varsigma,\nu}\breve{B}_\iota(k)\right\}_{r\sigma} = \sum_{\nu=1}^{s}\sum_{j=1}^{n}\sum_{i=1}^{n}\pi^{\iota}_{\varpi\nu}\varrho_{irj\sigma,\iota}\left(P^1_{\varsigma,\nu}\right)_{ij}. \tag{32}$$

Similarly, we can achieve that

$$\mathbb{E}\left\{\breve{B}^{\mathrm{T}}_\iota(k)P^2_{\varsigma,\nu}\breve{B}_\iota(k)\right\}_{r\sigma} = \sum_{\nu=1}^{s}\sum_{j=1}^{n}\sum_{i=1}^{n}\pi^{\iota}_{\varpi\nu}\varrho_{irj\sigma,\iota}\left(P^2_{\varsigma,\nu}\right)_{ij}$$

$$\mathbb{E}\left\{\breve{B}^{\mathrm{T}}_\iota(k)P^3_{\varsigma,\nu}\breve{B}_\iota(k)\right\}_{r\sigma} = \sum_{\nu=1}^{s}\sum_{j=1}^{n}\sum_{i=1}^{n}\pi^{\iota}_{\varpi\nu}\varrho_{irj\sigma,\iota}\left(P^3_{\varsigma,\nu}\right)_{ij}. \tag{33}$$

Considering the partition matrices $P_{\iota,\varpi}$, $W_{\iota,\varpi}$ in (29), and the conditions (32) and (33) of the stochastic terms, the inequalities in (27) guarantee that the conditions in (24) hold by defining $\mathcal{K}_{\iota,\varpi} \triangleq W^2_{\iota,\varpi}K_{\iota,\varpi}$, that is the conditions (27) guarantee that the augmented system (11) is stochastically stable with H_∞ performance γ. ■

Remark 4: The MATLAB tool box does not restrict the number of the decision variables for the LMIs, that is, the LMIs can contain thousands of decision variables as long as the computer capacity can support them. The number of the decision variable of (27) is $5smn^2 + (sm + 2)n$. Therefore, the estimator can be designed based on Theorem 3, if the dimension of the NNs is not too large.

V. Numerical Example

An example is employed to demonstrate the developed results for the stochastic NNs with uncertain weight matrices and packet dropouts. The following two-periodic NNs are considered:

$$A_1 = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}, A_2 = \begin{bmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.75 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.2 & 0.3 & 0.23 \\ 0.1 & 0.1 & 0.22 \\ 0.2 & 0.2 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.12 & 0.2 & 0.1 \\ 0 & 0.2 & 0.1 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}$$

$$\breve{B}_1(\vartheta(k)) = \mathrm{diag}\{0.02\vartheta_1(k),\ 0.02\vartheta_1(k),\ 0.03\vartheta_2(k)\}$$

$$\breve{B}_2(\vartheta(k)) = \mathrm{diag}\{0.03\vartheta_1(k),\ 0.03\vartheta_1(k),\ 0.02\vartheta_2(k)\}$$

$$C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.15 & 0 & 0 \\ 0 & 0.1 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0 & 0 & 0.15 \end{bmatrix}$$

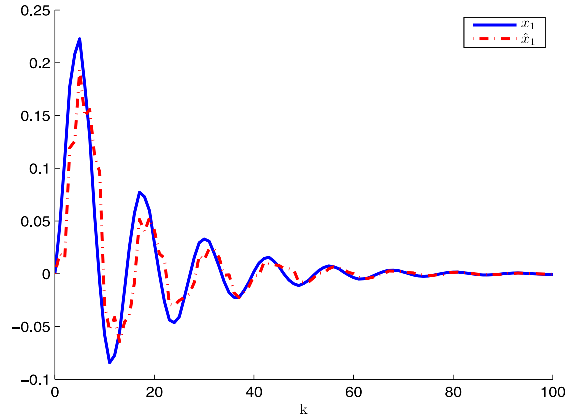


Fig. 2. Trajectories of $x_1(k)$ and its estimation.

$$E_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}, E_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.08 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}, S_2 = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}$$

where $\vartheta_1(k)$ and $\vartheta_2(k)$ are two correlated Gaussian white noises with expectation $\mathbb{E}\{\theta_1(k)\} = \mathbb{E}\{\theta_2(k)\} = 0$, the variance $\mathbb{C}\{\theta_1(k)\} = 1$ and $\mathbb{C}\{\theta_2(k)\} = 0.5$, and their covariance $\mathbb{C}\{\theta_1(k), \theta_2(k)\} = 1$.

Assume $\delta(k) \in \{1, 2\}$, which means that $\delta(k)$ is subjected to two states Markov chain with the transition probability matrices being the following forms:

$$\Pi^1 = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}, \quad \Pi^2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}.$$

The neuron activation functions are, respectively, supposed to be

$$g_1(x_1(k)) = \tanh(0.2x_1(k))$$
$$g_2(x_2(k)) = \tanh(-0.15x_2(k))$$
$$g_3(x_3(k)) = \tanh(0.2x_3(k))$$

which means

$$L_1 = \mathrm{diag}\{0, -0.15, 0\},\ L_2 = \mathrm{diag}\{0.2, 0, 0.3\}.$$

Based on Theorem 3, the gains of the estimators are

$$K_{11} = \begin{bmatrix} 0.4366 & -0.1756 \\ -0.0478 & 0.1935 \\ -0.1073 & 0.3683 \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} 0.4165 & -0.0287 \\ 0.1307 & 0.0171 \\ -0.0239 & 0.3800 \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} 0.6614 & -0.2896 \\ -0.2713 & 0.5006 \\ -0.0237 & 0.3932 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} 0.0607 & 0.1084 \\ 0.1884 & -0.1770 \\ -0.4751 & 0.7955 \end{bmatrix}$$

and the optimal H_∞ performance is $\gamma_{\min} = 0.3814$.

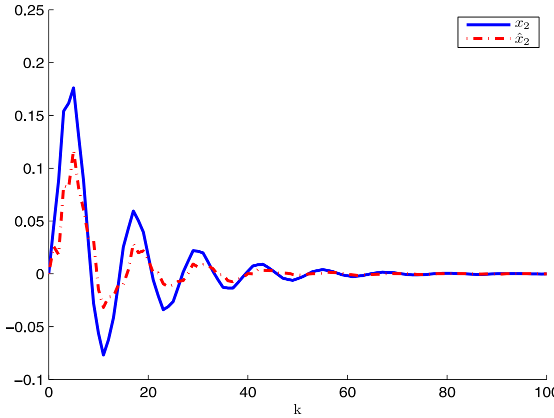


Fig. 3. Trajectories of $x_2(k)$ and its estimation.

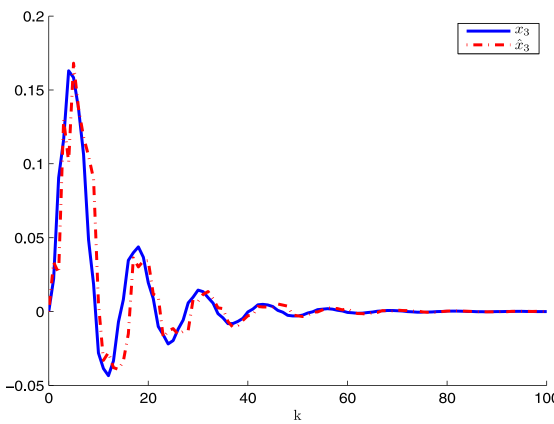


Fig. 4. Trajectories of $x_3(k)$ and its estimation.

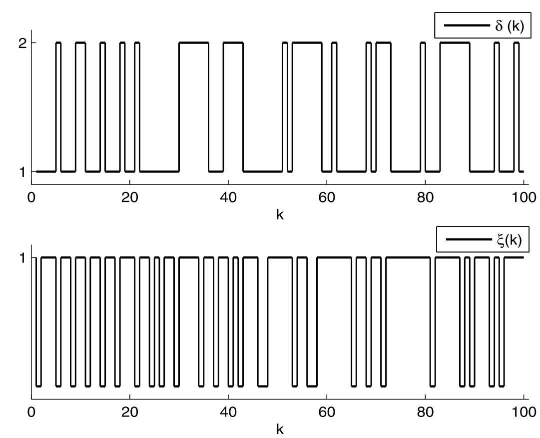


Fig. 5. Channel states and the packet dropouts.

Now, assume that the external disturbance $\mathbf{w}(k)$ is

$$\mathbf{w}(k) = \begin{bmatrix} \exp(-0.06k)\sin(0.5k) \\ \exp(-0.07k)\sin(0.5k) \\ \exp(-0.08k)\sin(0.5k) \end{bmatrix}$$

and initial conditions are $\mathbf{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$, and $\hat{\mathbf{x}}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$, respectively. The packet dropout rate is $\bar{\xi}_1 = 0.8$, and $\bar{\xi}_2 = 0.7$. Figs. 2–7 show the simulation results. The states of the NNs and their estimations are shown in Figs. 2–4. Fig. 5 depicts channel state $\delta(k)$ and the packer dropouts $\xi(k)$. Fig. 6 plots the noises of $\vartheta_1(k)$ and $\vartheta_2(k)$, respectively. The estimation error of the output $\mathbf{z}(k)$ is shown in Fig. 7. According to the noises $\mathbf{w}(k)$ and Fig. 7, we have $\sqrt{\sum_{k=0}^{\infty} \|\tilde{\mathbf{z}}(k)\|^2} = 0.4814 \leq \gamma\sqrt{\sum_{k=0}^{\infty} \|\mathbf{w}(k)\|^2} = 1.2609$, which implies that the H_∞ performance is satisfied.

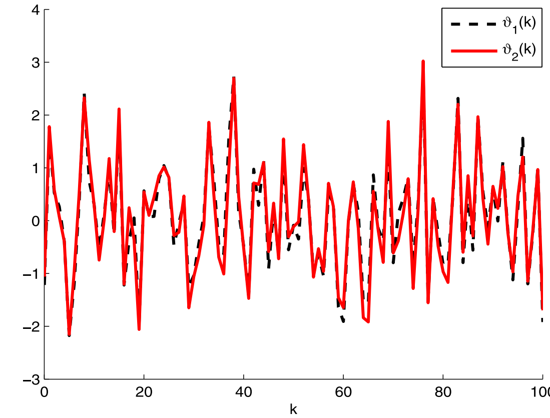


Fig. 6. Stochastic variables $\vartheta_1(k)$ and $\vartheta_2(k)$.

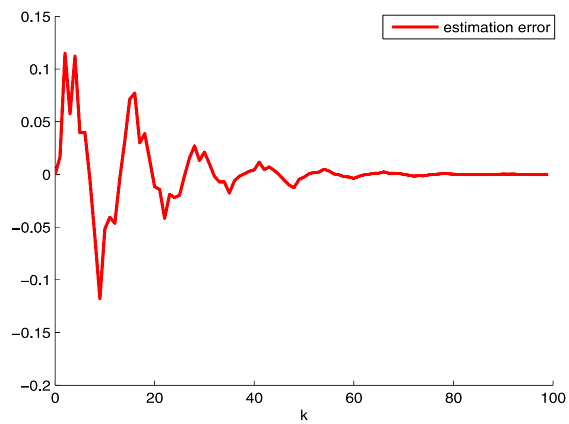


Fig. 7. Estimation error of the output $\mathbf{z}(k)$.

VI. Conclusion

The state estimator design issue has been studied for the periodic NNs with uncertain weight matrices, Markovian jump channel states, and packet dropouts. The stochastic parameter variables have been applied to characterize the random fluctuations of the connections to make the NNs more close to the practical model, and a Markov chain has been applied to describe the variation phenomenon of the channel states. By using the Lyapunov-like theory, a state estimator has been constructed such that the stochastic stability and the H_∞ performance of the augmented system are ensured. Finally, an example has been implemented to illustrate the validity of the proposed results. How to design the state estimator for the NNs with quantizer, event triggering mechanism, or unreliable sensors based on the Markov jump system analysis technique are our future works [44]–[47].

REFERENCES

[1] C. L. P. Chen, G.-X. Wen, Y.-J. Liu, and F.-Y. Wang, "Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 6, pp. 1217–1226, Jun. 2014.

[2] C. L. P. Chen, Y.-J. Liu, and G.-X. Wen, "Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems," *IEEE Trans. Cybern.*, vol. 44, no. 5, pp. 583–593, May 2014.

[3] Y.-J. Liu, J. Li, S. Tong, and C. L. P. Chen, "Neural network control-based adaptive learning design for nonlinear systems with full-state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 7, pp. 1562–1571, Jul. 2016.

[4] Y.-J. Liu, S. Tong, C. L. P. Chen, and D.-J. Li, "Neural controller design-based adaptive control for nonlinear MIMO systems with unknown hysteresis inputs," *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 9–19, Jan. 2016.

[5] Y. Xu, R. Lu, P. Shi, J. Tao, and S. Xie, "Robust estimation for neural networks with randomly occurring distributed delays and Markovian jump coupling," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2016.2636325.

[6] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, "Pinning control for synchronization of coupled reaction-diffusion neural networks with directed topologies," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 8, pp. 1109–1120, Aug. 2016.

[7] X. Li, R. Rakkiyappan, and G. Velmurugan, "Dissipativity analysis of memristor-based complex-valued neural networks with time-varying delays," *Inf. Sci.*, vol. 294, no. 10, pp. 645–665, 2015.

[8] Y. Chen and W. X. Zheng, "Stability analysis of time-delay neural networks subject to stochastic perturbations," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 2122–2134, Dec. 2013.

[9] X. Li and S. Song, "Impulsive control for existence, uniqueness, and global stability of periodic solutions of recurrent neural networks with discrete and continuously distributed delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 6, pp. 868–877, Jun. 2013.

[10] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Dissipativity analysis for discrete-time stochastic neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 3, pp. 345–355, Mar. 2013.

[11] X. Yang and D. W. C. Ho, "Synchronization of delayed memristive neural networks: Robust analysis approach," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3377–3387, Dec. 2016.

[12] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays," *IEEE Trans. Neural Netw.*, vol. 22, no. 10, pp. 1566–1575, Oct. 2011.

[13] X. Li, M. Bohner, and C.-K. Wang, "Impulsive differential equations: Periodic solutions and applications," *Automatica*, vol. 52, pp. 173–178, Feb. 2015.

[14] V. Dragan, "Optimal filtering for discrete-time linear systems with multiplicative white noise perturbations and periodic coefficients," *IEEE Trans. Autom. Control*, vol. 58, no. 4, pp. 1029–1034, Apr. 2013.

[15] P. Zhang, S. X. Ding, G. Z. Wang, and D. H. Zhou, "Fault detection of linear discrete-time periodic systems," *IEEE Trans. Autom. Control*, vol. 50, no. 2, pp. 239–244, Feb. 2005.

[16] X. Song, J. H. Park, and X. Yan, "Linear estimation for measurement-delay systems with periodic coefficients and multiplicative noise," *IEEE Trans. Autom. Control*, to be published, doi: 10.1109/TAC.2016.2621889.

[17] M. A. Rami and D. Napp, "Discrete-time positive periodic systems with state and control constraints," *IEEE Trans. Autom. Control*, vol. 61, no. 1, pp. 234–239, Jan. 2016.

[18] Y. Xu, R. Lu, P. Shi, H. Li, and S. Xie, "Finite-time distributed state estimation over sensor networks with Round–Robin protocol and fading channels," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2016.2635122.

[19] W.-H. Chen, S. Luo, and W. X. Zheng, "Generating globally stable periodic solutions of delayed neural networks with periodic coefficients via impulsive control," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2016.2552383.

[20] Z. Wang, D. W. C. Ho, and X. Liu, "State estimation for delayed neural networks," *IEEE Trans. Neural Netw.*, vol. 16, no. 1, pp. 279–284, Jan. 2005.

[21] L. Wang, Z. Wang, G. Wei, and F. E. Alsaadi, "Finite-time state estimation for recurrent delayed neural networks with component-based event-triggering protocol," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2016.2635080.

[22] Y. Chen and W. X. Zheng, "Stochastic state estimation for neural networks with distributed delays and Markovian jump," *Neural Netw.*, vol. 25, no. 1, pp. 14–20, 2012.

[23] X. Yin, Z. Li, L. Z. Zhang, and H. Minghao, "Distributed state estimation of sensor-network systems subject to Markovian channel switching with application to a chemical process," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2016.2632155.

[24] T. Wang, J. Qiu, H. Gao, and C. Wang, "Network-based fuzzy control for nonlinear industrial processes with predictive compensation strategy," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2016.2616904.

[25] Y. Xu, R. Lu, H. Peng, and S. Xie, "Filtering for fuzzy systems with multiplicative sensor noises and multi-density quantizer," *IEEE Trans. Fuzzy Syst.*, to be published, doi: 10.1109/TFUZZ.2017.2702119.

[26] D. Zhang, H. Song, and L. Yu, "Robust fuzzy-model-based filtering for nonlinear cyber-physical systems with multiple stochastic incomplete measurements," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2016.2551200.

[27] J.-G. Dong and L. Qiu, "Flocking of the Cucker—Smale model on general digraphs," *IEEE Trans. Autom. Control*, to be published, doi: 10.1109/TAC.2016.2631608.

[28] L. Zhang, Y. Zhu, P. Shi, and Y. Zhao, "Resilient asynchronous H_∞ filtering for Markov jump neural networks with unideal measurements and multiplicative noises," *IEEE Trans. Cybern.*, vol. 45, no. 12, pp. 2840–2852, Dec. 2015.

[29] B. Shen, Z. Wang, and H. Qiao, "Event-triggered state estimation for discrete-time multidelayed neural networks with stochastic parameters and incomplete measurements," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 5, pp. 1152–1163, May 2017, doi: 10.1109/TNNLS.2016.2516030.

[30] D. Ding, Z. Wang, H. Dong, and H. Shu, "Distributed H_∞ state estimation with stochastic parameters and nonlinearities through sensor networks: The finite-horizon case," *Automatica*, vol. 48, no. 8, pp. 1575–1585, 2012.

[31] Y. Luo *et al.*, "Globally optimal multisensor distributed random parameter matrices Kalman filtering fusion with applications," *Sensors*, vol. 8, no. 12, pp. 8086–8103, 2008.

[32] W. Zhang, Y. Huang, and H. Zhang, "Stochastic H_2/H_∞ control for discrete-time systems with state and disturbance dependent noise," *Automatica*, vol. 43, no. 3, pp. 513–521, 2007.

[33] N. Xiao, L. Xie, and L. Qiu, "Feedback stabilization of discrete-time networked systems over fading channels," *IEEE Trans. Autom. Control*, vol. 57, no. 9, pp. 2176–2189, Sep. 2012.

[34] S. Arik, "New criteria for global robust stability of delayed neural networks with norm-bounded uncertainties," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 6, pp. 1045–1052, Jun. 2014.

[35] H. Huang, G. Feng, and J. Cao, "Robust state estimation for uncertain neural networks with time-varying delay," *IEEE Trans. Neural Netw.*, vol. 19, no. 8, pp. 1329–1339, Aug. 2008.

[36] C. Li, J. Chen, and T. Huang, "A new criterion for global robust stability of interval neural networks with discrete time delays," *Chaos Solitons Fractals*, vol. 31, no. 3, pp. 561–570, 2007.

[37] Y. Liu, Z. Wang, and X. Liu, "Global exponential stability of generalized recurrent neural networks with discrete and distributed delays," *Neural Netw.*, vol. 19, no. 5, pp. 667–675, 2006.

[38] P. Sadeghi, R. A. Kennedy, P. B. Rapajic, and R. Shams, "Finite-state Markov modeling of fading channels—A survey of principles and applications," *IEEE Signal Process. Mag.*, vol. 25, no. 5, pp. 57–80, Sep. 2008.

[39] A. G. Marques, X. Wang, and G. B. Giannakis, "Minimizing transmit power for coherent communications in wireless sensor networks with finite-rate feedback," *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4446–4457, Sep. 2008.

[40] R. Lu, Y. Xu, and R. Zhang, "A new design of model predictive tracking control for networked control system under random packet loss and uncertainties," *IEEE Trans. Ind. Electron.*, vol. 63, no. 11, pp. 6999–7007, Nov. 2016.

[41] L. Zhang and E. K. Boukas, "Mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 6, pp. 1462–1467, 2009.

[42] G. Wei, G. Feng, and Z. Wang, "Robust H_∞ control for discrete-time fuzzy systems with infinite-distributed delays," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 1, pp. 224–232, Feb. 2009.

[43] C. A. C. Gonzaga and O. L. V. Costa, "Stochastic stabilization and induced l_2-gain for discrete-time Markov jump Lur'e systems with control saturation," *Automatica*, vol. 50, no. 9, pp. 2397–2404, 2014.

[44] X. Li and R. Rakkiyappan, "Delay-dependent global asymptotic stability criteria for stochastic genetic regulatory networks with Markovian jumping parameters," *Appl. Math. Model.*, vol. 36, no. 4, pp. 1718–1730, 2012.

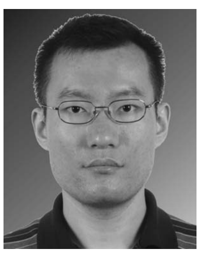
[45] Z.-G. Wu, P. Shi, Z. Shu, H. Su, and R. Lu, "Passivity-based asynchronous control for Markov jump systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 2020–2025, Apr. 2017.

[46] J. Zhang, L. Ma, and Y. Liu, "Passivity analysis for discrete-time neural networks with mixed time-delays and randomly occurring quantization effects," *Neurocomputing*, vol. 216, pp. 657–665, Dec. 2016.

[47] Q. Li, B. Shen, Y. Liu, and F. E. Alsaadi, "Event-triggered H_∞ state estimation for discrete-time stochastic genetic regulatory networks with Markovian jumping parameters and time-varying delays," *Neurocomputing*, vol. 174, pp. 912–920, Jan. 2016.

Yong Xu was born in Zhejiang Province, China, in 1983. He received the B.S. degree from Nanchang Hangkong University, Nanchang, China, in 2007, the M.S. degree from Hangzhou Dianzi University, Hangzhou, China, in 2010, and the Ph.D. degree from Zhejiang University, Hangzhou, in 2014.

In 2013, He was an visiting internship student with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong, for six months. He is currently an Associate Professor with the School of Automation, Guangdong University of Technology, Guangzhou, China. His current research interests include networked control systems, estimation, and stochastic systems.



Zhuo Wang was born in Handan, China, in 1983. He received the B.E. degree in automation from Beihang University, Beijing, China, in 2006, and the Ph.D. degree in electrical and computer engineering from University of Illinois at Chicago, Chicago, IL, USA, in 2013.

He was a Post-Doctoral Fellow with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada, from 2013 to 2014. He was a Research Assistant Professor with the Fok Ying Tung Graduate School, Hong Kong University of Science and Technology, Hong Kong, from 2014 to 2015. He is currently a Professor and a Ph.D. Instructor with the School of Instrumentation Science and Optoelectronics Engineering, Beihang University.

Prof. Wang was a recipient of the Thousand Talents Program for Distinguished Young Scholars by the Organization Department of the CPC Central Committee, and the 100 Talents Program by Beihang University, in 2015. He is an Associate Editor of the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, *Control Theory and Applications*, and *Pattern Recognition and Artificial Intelligence*. He is a member of the Adaptive Dynamic Programming and Reinforcement Learning Technical Committee of the IEEE Computational Intelligence Society, the Data Driven Control, Learning and Optimization Professional Committee of CAA, and the Fault Diagnosis and Safety for Technical Processes Professional Committee of CAA.

Deyin Yao received the M.S. degree in control theory from Research Institute of Automation, Bohai University, Jinzhou, China, in 2016. He is currently pursuing the Ph.D. degree with the School of Automation, Guangdong University of Technology, Guangzhou, China.

His current research interests include robust control, sliding mode control, and neural control.

Renquan Lu received the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, China, in 2004.

He was a Research Fellow with the School of Electrical Engineering and Computer Science, University of Newcastle, Callaghan, NSW, Australia. He is currently a Full Professor with the School of Automation, Guangdong University of Technology, Guangzhou, China. His current research interests include robust control, networked control systems, and complex systems.

Prof. Lu was a recipient of the National Science Fund for Distinguished Young Scientists of China in 2014, and the New Century Excellent Talents in University of Ministry of Education of China in 2010.

Chun-Yi Su (SM'99) received the Ph.D. degree in control engineering from the South China University of Technology, Guangzhou, China, in 1990.

He joined Concordia University, Montreal, QC, Canada, in 1998, after a seven-year stint with the University of Victoria, Victoria, BC, Canada. He is currently with the School of Automation, Guangdong University of Technology, Guangzhou, on leave from Concordia University. He has authored or co-authored over 300 publications in journals, book chapters, and conference proceedings. His current research interests include the application of automatic control theory to mechanical systems and control of systems involving hysteresis nonlinearities.

Dr. Su has served as an Associate Editor of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, and the *Journal of Control Theory and Applications*. He has been on the Editorial Board of 18 journals, including the *IFAC Journal of Control Engineering Practice and Mechatronics*.