The First International Conference on Mechanical System Dynamics (the 1st ICMSD)
August 12-18, 2022, Nanjing, China
Paper No. ICMSD-000502

Location and Inventory Optimization Model of Spare Parts Warehouse Based on Uncertainty Theory

Yi Yang[1,3], Wei Liu[1], Meilin Wen[1*], Yong Qin[2*], Tingting Zeng[1], Siyu Huang[1]

[1]School of Reliability and Systems Engineering, Beihang University, Beijing, China; yiyang@buaa.edu.cn(Yi Yang), liuwei1@buaa.edu.cn(Wei Liu), sy2014101@buaa.edu.cn(Tingting Zeng) , huangsiyu@buaa.edu.cn(Siyu Huang)
[2]State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing, China;
[3]Peng Cheng Laboratory, Shenzhen, China
**email address of corresponding author , wenmeilin@buaa.edu.cn(Meilin Wen) , yqin@bjtu.edu.cn (Yong Qin);*

Keywords: SPARE PARTS MANAGEMENT, LOCATION AND INVENTORY CONTROL, UNCERTAIN PROGRAMMI NG MODEL.

Abstract

In equipment integrated logistics support, support activities and support resources have a vital impact on the availability of equipment system. Spare parts warehouse location and inventory strategy will directly affect spare parts supply, and finally affect the system operation. Therefore, this paper establishes an optimization model of spare parts warehouse location considering inventory strategy. In order to deal with the epistemic uncertainty of spare parts demand, the uncertainty theory is introduced. The model aims at reducing the comprehensive support cost, adds the supportability parameters to the model constraints, and optimizes the inventory control and spare parts warehouse location as a whole. The genetic algorithm is used to solve the model, and the optimal location scheme and inventory strategy meeting the supportability constraints are obtained. Based on the proposed model and calculation results, the accuracy of equipment support can be improved and the support cost can be reduced as much as possible on the premise of ensuring equipment availability.

1 Introduction

Reasonable spare parts inventory can minimize the risk of downtime and ensure equipment availability, while excessive spare parts inventory will lead to unnecessary inventory costs. The location of spare parts warehouse affects the supply of spare parts, and the implementation of spare parts inventory strategy will affect the spare parts inventory. Therefore, inventory and location should be optimized as a whole to achieve a balance between cost and supportability.

The demand for spare parts comes from the process of equipment operation and maintenance[1], and the participation and decision of humans in spare parts management brings epistemic uncertainty to the demand for spare parts. Different types of uncertainty require different treatment methods[2]. In classical probability theory, the probability distribution of uncertain factors can be estimated according to a large number of historical data. However, in many cases, the sample size is not enough to estimate the probability distribution, so other theories have been put forward and applied. In 1965, Zadeh[3] put forward the fuzzy set theory, which uses the membership function to quantify the fuzziness. Song et al.[4] established a location optimization model, which has an optimal goal to maximize profit and market response efficiency under the condition of fuzzy demand. Ilbahar et al.[5] used a fuzzy linear programming model to deal with the uncertainty in the obtained amount of biomass, and finally found the most suitable locations. Pawlak[6] proposed rough set theory in 1982, which is an effective tool to deal with imprecise and incomplete information. Chen[7] developed a data mining framework based on rough set theory to support location selection decision.

However, probability theory and fuzzy mathematics are not enough to cover all non-deterministic phenomena, especially, the epistemic uncertainty. Due to the lack of sample data and in order to deal with human cognitive limitations, Liu[8] proposed an uncertainty theory, which can describe the imprecise information from experts or human experience, so as to deal with epistemic uncertainty, and has been applied in many fields from then on, such as optimal control problem[9], structural reliability[10], portfolio selection problem[11], stock investment[12]. In order to deal with the non-deterministic factor vertex demand, Gao[13] employed the uncertainty theory to facility location problems. Zhang et al.[14] have solved the location problem of emergency service facilities under uncertainty based on uncertainty theory.

Therefore, this paper introduces the uncertainty theory, regards the spare parts demand as an uncertain variable which follows the uncertain normal distribution. The optimization model of spare parts warehouse location and inventory control is established and solved by genetic algorithm.

2. Preliminaries

The uncertainty theory was proposed and developed by Liu[8], this section mainly introduces the basic definition of the uncertainty theory.

Definition 1.(*Uncertain Measure*)[8]

In uncertainty theory, uncertain measure $\mathcal{M}\{\Lambda\}$ was defined on the $\sigma-$algebra, which is used to describe the belief degree that an uncertain event Λ may happen. In order to rationally deal with belief degrees, the following three axioms are suggested, and the set function $\mathcal{M}$ is called an uncertain

measure if it satisfifies the normality, duality, and subadditivity axioms.
Axiom 1. (*Normality Axiom*[8]) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.
Axiom 2. (*Duality Axiom*[8]) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.
Axiom 3. (*Subadditivity Axiom*[8]) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\{\cup_{i=1}^{\infty} \Lambda_i\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\} \quad (2.1)$$

Definition2. (*Uncertain Variable*)[8]
An uncertain variable is a function ξ from an uncertainty space to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set B.
Definition3. (*Uncertainty Distribution*)[8]
The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\} \quad (2.2)$$

for any real number x.
Definition4. (*Normal Uncertainty Distribution*)[8]
An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, x \in \Re \quad (2.3)$$

denoted by $N(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$.
Definition5. (*Expected*)[8]
For an uncertain variable ξ, its expected value is defined by

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge x\}\, dx - \int_{-\infty}^{0} \mathcal{M}\{\xi \le x\} dx \quad (2.4)$$

3 Problem Description

In order to guarantee the spare parts of the equipment system more efficiently, it is necessary to select several from the existing sites as the spare parts warehouse, which are responsible for the spare parts supply of the remaining demand sites. Besides, the inventory strategy will affect the supply of spare parts, which also needs to be determined. In order to achieve the optimal cost control while meeting the requirements of supportability, the optimization model of spare parts warehouse location and inventory control is established.

3.1 Model Hypothesis

(1) The demand of spare parts for each site is an uncertain variable, which follows the uncertain normal distribution;
(2) Supposing that the spare parts warehouse implements (r, Q) inventory strategy[15], which means that a batch of size Q is triggered when the inventory level declines to r;
(3) Assuming that all the spare parts on the equipment are of the same importance, and any failure will lead to the shutdown of the equipment system.

3.2 Symbol Definition

The basic symbol definition of this paper are as follows:

Table 1 Symbol Definition and Meanings

Symbol	Definition	Remarks
$\boldsymbol{i}$	Spare parts demand site	$i = 1,2,\ldots,I$
$\boldsymbol{j}$	Candidate spare parts warehouse	$j = 1,2,\ldots,J$
$\boldsymbol{L}$	Order lead time of spare parts warehouse	
$\boldsymbol{s_i}$	The quantity of spare parts supplied by the spare parts warehouse j to the demand site i	$i = 1,2,\ldots,I$ $j = 1,2,\ldots,J$
$\boldsymbol{\xi_i}$	Spare parts demand of demand site i	$i = 1,2,\ldots,I$
$\boldsymbol{y_j}$	Due inventory level of spare parts warehouse j, which obeys a uniform distribution of $\{r_j + 1, r_j + 2, \ldots, r_j + Q_j\}$	$j = 1,2,\ldots,J$
$\boldsymbol{d_{ij}}$	The distance between the demand site i and the spare parts warehouse j	$i = 1,2,\ldots,I$ $j = 1,2,\ldots,J$
$\boldsymbol{f_j}$	Construction cost of spare parts warehouse at location j	$j = 1,2,\ldots,J$
$\boldsymbol{r_j}$	Reorder point of spare parts warehouse j	$j = 1,2,\ldots,J$
$\boldsymbol{Q_j}$	Fixed order quantity of spare parts warehouse j	$j = 1,2,\ldots,J$
$\boldsymbol{h}$	The unit storage cost of spare parts warehouse	
$\boldsymbol{g}$	Loss of unit spare part shortage	
$\boldsymbol{F}$	The cost of one order	
$\boldsymbol{N_i}$	Equipment quantity of demand site i	
$\boldsymbol{Z}$	Number of single installation of spare parts	
$\boldsymbol{A_i}$	Lower limit of equipment supply availability for site i	
$\boldsymbol{A_0}$	System availability	
$\boldsymbol{X_j}$	$X_j = \begin{cases} 1 & \text{site } j \text{ is warehouse} \\ 0 & \text{site } j \text{ is not warehouse} \end{cases}$	$j = 1,2,\ldots,J$
$\boldsymbol{Y_{ij}}$	$Y_{ij} = \begin{cases} 1 & site\ i \text{ is supplied by } j \\ 0 & site\ i \text{ is } not \text{ supplied by } j \end{cases}$	$i = 1,2,\ldots,I$ $j = 1,2,\ldots,J$

4 Location and Inventory Uncertain Optimization Model

This chapter analyzes the cost of spare parts warehouse location and spare parts supply, takes the minimization of support cost as the goal of the model, and the support requirements of the system are taken as the constraints of the model.

4.1 Cost Analysis

(1) Inventory Cost

The inventory cost includes reorder cost, inventory holding cost and shortage loss cost[16]. The inventory holding cost is the additional inventory holding cost, and the shortage cost refers to the loss caused by the shortage of spare parts. Besides, the inventory cost is affected by the inventory strategy. Under the continuous inventory strategy (r, Q), taking an order lead time as the unit time, the due inventory level after the order arrives obeys the uniform distribution of $\{r+1, r+2, \ldots, r+Q\}$, supposing that the demand is generated at the same time and the total spare parts demand of the warehouse j is $\sum_{i \in I} \xi_i Y_{ij}$ exactly[17]. When the inventory level of warehouse j is smaller than the total demand of its corresponding sites, out of stock will occur, and the shortage penalty cost of warehouse j is

$$C_S = g\left\{\sum_{i \in I} (E(\xi_i) - s_i)^+ Y_{ij}\right\} \tag{4.1}$$

Where g in formula (4.1) represents the loss caused by a shortage. When the inventory level of warehouse j is greater than the total demand of its corresponding sites, there will be additional inventory holding costs. The inventory holding cost of warehouse j is

$$C_L = \frac{1}{Q_j}\left\{\sum_{y_j=r_j+1}^{r_j+Q_j} \left(h\left(y_j - \sum_{i \in I} E(\xi_i) Y_{ij}\right)^+\right)\right\} \tag{4.2}$$

Where h in formula (4.2) represents the extra holding cost caused by one overflow. In a lead time, the inventory cost of j includes order cost F, inventory holding cost C_L and shortage penalty cost C_S. And the total inventory cost of all the warehouses is represented by C_H.

$$C_H = \sum_{j \in J} X_j \{F + C_S + C_L\} \tag{4.3}$$

(2) Construction Cost

Assuming that the construction cost of all warehouses is only related to the number of spare parts warehouses, so it is expressed as

$$C_F = \sum_{j \in J} f_j X_j \tag{4.4}$$

(3) Transportation Cost

It is assumed that the transportation cost is only related to the distance between sites and transportation volume, so the total transportation cost of spare parts is

$$C_T = \sum_{j \in J} \sum_{i \in I} d_{ij} s_i Y_{ij} \tag{4.5}$$

(4) Total Cost

The total cost C is the sum of inventory cost C_H, construction cost C_F, and transportation cost C_T, which is regarded as the objective function.

$$C = C_H + C_F + C_T \tag{4.6}$$

4.2 Uncertain Constraints

(1) Spare Parts Supportability

The inventory level of warehouse j may not be able to meet the demand of all corresponding demand sites, which will cause a shortage.

$$E(\xi_i) \le s_i, \; i = 1,2,\ldots,n \tag{4.7}$$

(2) Supply Availability

Sherbrooke[18] proposed the formula of supply availability, which is as follows,

$$A_s = \prod_{i=1}^{I} \left(1 - \frac{EBO_i}{NZ_i}\right)^{Z_i} \tag{4.8}$$

Where I is the number of unit types, N is the number of equipment, Z_i is the number of units used in the equipment. The supply availability of site i should be no less than A_i. Where N_i is the number of equipment in site i, Z is the installed number of the spare part in an equipment.

$$\left[1 - E(\xi_i) - s_i) \frac{1}{N_i Z}\right]^Z \ge A_i, \; i = 1,2,\ldots,n \tag{4.9}$$

Besides, the system availability should be no less than A_0.

$$\left(\sum_{i=1}^{n} N_i \left[1 - (E(\xi_i) - s_i) \frac{1}{N_i Z}\right]^Z / \sum_{i=1}^{n} N_i\right) \ge A_0, \; i = 1,2,\ldots,n \tag{4.10}$$

(3) Spare Parts Allocation Constraints

When the inventory strategy is determined, the total allocation of warehouse j to the responsible demand sites cannot exceed the nominal maximum inventory level of j,

$$\sum_{i \in I} s_i Y_{ij} \le \sum_{j \in J} (r_j + Q_j) \tag{4.11}$$

In addition, it is assumed that each spare parts demand point can only be served by one spare parts warehouse, and X_j and Y_{ij} are binary variables. Based on the above analysis, a location and inventory uncertain optimization model is proposed, in which the demand of spare parts ξ_i is an uncertain variable, which follows an uncertain normal distribution with parameters e and σ.

$$
\begin{cases}
\min \sum_{j\in J} f_j X_j + \sum_{j\in J}\sum_{i\in I} d_{ij} s_i Y_{ij} + \sum_{j\in J} F X_j \\
+\sum_{j\in J} X_j \cdot g\left\{\sum_{i\in I}\left(E(\xi_i)-s_i\right)^+ Y_{ij}\right\} \\
+\sum_{j\in J} X_j \cdot \left\{\frac{1}{Q_j}\left\{\sum_{y_j=r_j+1}^{r_j+Q_j}\left(h\left(y_j-\sum_{i\in I}E(\xi_i)Y_{ij}\right)^+\right)\right\}\right\} \\
\text{subject to}: \\
E(\xi_i)\le s_i,\ i=1,2,...,I \\
\left[1-\frac{E(\xi_i)-s_i}{N_i Z}\right]^Z \ge A_i,\ i=1,2,...,I \\
\sum_{i=1}^{I} N_i\left[1-\frac{E(\xi_i)-s_i}{N_i Z}\right]^Z \Big/ \sum_{i=1}^{I} N_i \ge A_0 \\
\sum_{i\in I} s_i Y_{ij} \le \sum_{j\in J}\left(r_j+Q_j\right),\ j=1,2,...,J, \\
\sum_{j\in J} Y_{ij}=1,\ i=1,2,...,I, \\
Y_{ij}\le X_j,\ i=1,2,...,I,\ j=1,2,...,J, \\
X_j\in\{0,1\},\ j=1,2,...,J, \\
Y_{ij}\in\{0,1\},\ i=1,2,...,I,\ j=1,2,...,J.
\end{cases} \tag{4.12}
$$

5 Case Study

A case study is designed to verify the practicability of the model. Table 1 shows the specific location information and some other related parameters of the existing ten sites.

Table 2 Location Information and Parameter Setting of Each Site

Site	**Coordinate**	N	Z	f	F	g	h	e	σ	A_i
Site 1	（96,44）	5	3	100	1	2	1	90	35	0.9
Site 2	（18,98）	6	3	100	1	2	1	70	20	0.9
Site 3	（16,21）	8	3	100	1	2	1	100	25	0.9
Site 4	（17,53）	5	3	200	1	2	1	85	30	0.9
Site 5	（74,67）	7	3	200	1	2	1	65	40	0.9
Site 6	（19,49）	6	3	300	1	2	1	70	40	0.9
Site 7	（3,8）	3	3	300	1	2	1	83	20	0.9
Site 8	（54,73）	8	3	200	1	2	1	76	20	0.9
Site 9	（64,48）	3	3	100	1	2	1	85	30	0.9
Site 10	（98,58）	7	3	300	1	2	1	78	20	0.9

Genetic algorithm is very suitable and widely used for solving complex nonlinear programming problems[19-22]. Therefore, a genetic algorithm is adopted to solve the model. Table 2 shows the calculation results, namely, transport quantity from warehouse to demanding site, and inventory strategy performed in warehouses.

Table 3 Supply Scheme & Inventory Strategy

Demanding Sites	Site 1	Site 2		Site 4	
s_i	90	70		85	
Warehouses			Site 3		Site 5
(r,Q)			(122,52)		(108,52)
Corresponding Warehouse	Site 9	Site 5		Site 3	
Demanding Sites	Site 6	Site 7	Site 8		Site 10
s_i	70	83	76		78
Warehouses				Site 9	
(r,Q)				(115,21)	
Corresponding Warehouse	Site 5	Site 3	Site 3		Site 3

6 Conclusion

This paper has established an expected value optimization model for spare parts warehouse location and inventory control, which tries to make availability and supportability cost balanced. Considering about the epistemic uncertainty of spare parts demand derived from human participation and cognitive limitations, the uncertainty theory was introduced into the model, which can make up for the deficiency of other traditional mathematical theories. The location scheme and inventory strategy of supportability and economical balance were solved by genetic algorithm. This study has given a more reasonable spare parts management and inventory decision-making scheme, which can be used to guide the decision-making of equipment comprehensive support.

7 Acknowledgements

This work was supported by National Science Foundation of China under Grants 61871013, 92167110, 62073009, the State Key Laboratory of Rail Traffic Control and Safety (Contract No.RCS2021K004), Beijing Jiaotong University and the National Defense Technology Basic Research Foundation.

8 References

[1] Choi, B., Suh, J.H.: 'Forecasting Spare Parts Demand of Military Aircraft: Comparisons of Data Mining Techniques and Managerial Features from the Case of South Korea', Sustainability, 2020, 12, (15), pp. 6045

[2] Hu, L.H., Kang, R., Pan, X., et al.: 'Uncertainty Expression and Propagation in the Risk Assessment of Uncertain Random System', Ieee Systems Journal, 2021, 15, (2), pp. 1604-1615

[3] Zadeh, L.A.: 'Fuzzy sets', Information &Control, 1965, 8, (3), pp. 338-353

[4] Song, X.Y., Ge, P., Wan, Z.Y., et al.: 'Fuzzy Multi-target Distribution Center Location and Inventory Setting

Model and Simulation Solution', the 7th International Conference on Industrial Technology and Management (ICITM 2018), Oxford, England, March 2018, pp. 314-319

[5] Ilbahar, E., Kahraman, C., Cebi, S.: 'Location Selection for Waste-to-energy Plants by Using Fuzzy Linear Programming', Energy, 2021, 234, pp. 121189

[6] Pawlak, Z.: 'Rough Sets', International Journal of Computer & Information Sciences, 1982, 11, pp. 341-356

[7] Chen, L.F., Tsai, C.T.: 'Data Mining Framework Based on Rough Set Theory to Improve Location Selection Decisions: A Case Study of a Restaurant Chain', Tourism Management, 2016, 53, pp. 197-206

[8] Liu, B.D.: 'Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty', (Springer Berlin Heidelberg, 2010)

[9] Chen, X., Zhu, Y.G., Sheng, L.X.: 'Optimal Control for Uncertain Stochastic Dynamic Systems with Jump and Application to an Advertising Model', Applied Mathematics and Computation, 2021, 407, pp. 126337

[10] Wang, P.D., Zhang, J.G., Zhai, H., et al.: 'A New Structural Reliability Index Based on Uncertainty Theory', Chinese Journal of Aeronautics, 2017, 30, (4), pp. 1451-1458

[11] Zhu, Y.G.: 'Uncertain Optimal Control with Application to a Portfolio Selection Model', Cybernetics and Systems, 2010, 41, (7), pp. 535-547

[12] Chen, X.W., Liu, Y.H., Ralescu, D.A.: 'Uncertain Stock Model with Periodic Dividends', Fuzzy Optimization and Decision Making, 2013, 12, (1), pp. 111-123

[13] Gao, Y.: 'Uncertain Models for Single Facility Location Problems on Networks', Applied Mathematical Modelling, 2012, 36, (6), pp. 2592-2599

[14] Zhang, B., Peng, J., Li, S.G.: 'Covering Location Problem of Emergency Service Facilities in An Uncertain Environment', Applied Mathematical Modelling, 2017, 51, pp. 429-447

[15] Shenas, N.Y., Jahromi, A.E., Yazdi, M.M.: 'An Efficient Procedure for Computing an Optimal (R, Q) Policy in Continuous Review Systems with Poisson Demands and Constant Lead Time', Scientia Iranica Transaction E-Industrial Engineering, 2009, 16, (2), pp. 128-137

[16] Wang, Q., Wu, J., Zhao, N.G., et al.: 'Inventory Control and Supply Chain Management: A green growth perspective', Resources Conservation and Recycling, 2019, 145, pp. 78-85

[17] Federgruen, A., Zheng, Y.S.: 'An Efficient Algorithm for Computing an Optimal (r, Q) Policy in Continuous Review Stochastic Inventory Systems', Operations Research, 1992, 40, (4), pp. 808-813

[18] Sherbrooke, C.C.: 'Optimal Inventory Modeling of Systems', International Series in Operations Research & Management Science, (Springer, Boston, MA, USA, 2004)

[19] Shu, J., Ma, Q.A., Li, S.J.: 'Integrated Location and Two-echelon Inventory Network Design under Uncertainty', Annals of Operations Research, 2010, 181, (1), pp. 233-247

[20] Li, Y.H., Guo, H., Zhang, Y.: 'An Integrated Location-inventory Problem in a Closed-loop Supply Chain with Third-party Logistics', International Journal of Production Research, 2018, 56, (10), pp. 3462-3481

[21] Wang, L., Qu, H., Chen, T., et al.: 'An Effective Hybrid Self-adapting Differential Evolution Algorithm for the Joint Replenishment and Location-inventory Problem in a Three-level Supply Chain', Scientific World Journal, 2013, 2013, pp. 270249

[22] Celebi, D.: 'Inventory Control in a Centralized Distribution Network Using Genetic Algorithms: A Case Study', Computers & Industrial Engineering, 2015, 87, pp. 532-539