Instantaneous Availability Analysis of Maintenance Process Based on Semi-Markov Model

Yi Yang, Tingting Zeng, Siyu Huang, and Wei Liu

1 Introduction

In the using process of equipment, availability describes the ability of equipment to perform the specified tasks normally at any time. It can also measure operational readiness and mission sustainability. Current researches on availability mainly focus on steady-state availability application and instantaneous availability modeling in complex systems. Steady-state availability reflects the usability when the system reaches a stable operation after a long running time. However, in the early stage of equipment use, it fails to describe both the fluctuation of operational capability and the variety of availability. In addition, the analysis of the initial fluctuation of equipment availability can effectively reflect the real-time performance of equipment.

There are three main factors that affect the availability of equipment, namely, reliability, maintainability, and supportability [1–4]. The influence of reliability and maintainability can be reduced by reasonable design of development period. However, if the system fails to attain support resource after failure, it will delay the maintenance action, induce support delay, and impact the availability. The concept of support delay was presented in 1973 [5]. As the lucubration of support delay on system availability, support delay time comes into focus [6, 7]. Reference [8] utilized the Markov process and Laplace transform to obtain an expression for steady-state availability after establishing a model that contains the time of prevention and maintenance. Reference [9] used the Markov renewal process theory and total probability decomposition technique to obtain the steady-state availability of the system. There are many available models that consider support delay, but

Y. Yang · T. Zeng · S. Huang (✉) · W. Liu
Beihang University, Beijing, China

D. Tang et al. (eds.), *Mobile Wireless Middleware, Operating Systems and Applications*, EAI/Springer Innovations in Communication and Computing,
https://doi.org/10.1007/978-3-030-98671-1_7

majority of them concentrate on steady-state availability. Meanwhile, researches of support delay time usually center on causing by a single support resource, such as equipment delay, queue waiting, and others [10, 11]. The study caused by multiple support factors is rarely being launched. Therefore, this paper will comprehensively consider the impact of multiple factors on instantaneous availability of support process.

Support delay is mainly caused by three factors: equipment, spares, and personnel. The system has several possible states while waiting for maintenance. The available modeling methods of multistate system are divided into four categories, namely, Boolean extended model method, Markov model method, general generation function method, and Monte Carlo simulation method. Markov model method can effectively characterize the state transformation process of the system and deconstruct transition speciality of multistate. Reference [12] introduced a special method called LZ transformation to analyze instantaneous availability in Markov process of discrete state and continuous time. During support process, the state stationary time does not fully satisfy exponential distribution, which is a condition of standard Markov process. For this situation, researchers use simulation methods to study it [13]. In Reference [14], an analytical and probabilistic availability model for periodical inspection system is proposed by a new recursive algorithm, which can achieve limiting average availability and instantaneous availability of periodical inspection system under arbitrary lifetime and repair-time distributions. In Reference [15–17], by using traditional empirical modeling methods, they established availability evaluation models of complex systems. However, facing the transition process between different states, traditional model is unsuitable. Meanwhile, it is more universal to implement semi-Markov process in the modeling of support delay process that obeys random distribution. At present, researches on multistate system reliability modeling with semi-Markov transition are scarce [18–20]. So the application of support delay construction remains to be studied.

In this paper, based on semi-Markov method, we established a more universal instantaneous availability analysis model, which, considering delay factors of support personnel, supports equipment and spares. The analytic expression of instantaneous availability is obtained by Fourier transform. Besides, we investigated the change of instantaneous availability over time.

The paper is organized as follows. In Sect. 2, we present related definitions of semi-Markov model. In Sect. 3, the instantaneous availability model of repairable system considering support delay is proposed, and a numerical example is given in Sect. 4.

2 Preliminaries

The assumption that state performance space is discrete while time course space is continuous is more corresponding to engineering reality of state transition. So we only based on continuous semi-Markov model for mathematical interpretation.

We consider a system S With m possible states, m is a finite natural number. The set of all possible states is noted as $I = \{1, 2, \cdots, m\}$. For discrete time $n \in N_0$, $N_0 = \{0, 1, 2, \cdots\}$, the initial state of S at time 0 is represented by random variable J_0. The system S stays a random length of time X_1 in the initial state, then enters the next state J_1 to stay for a random length of time X_2 before going into J_2, and so on. For discrete time, the sequence $(J_n, n \geq 0)$ represents the successive states of S. The sequence $(X_n, n \geq 0)$ gives the successive sojourn time. The two-dimensional stochastic process in discrete time is called a positive $(J - X)$ process:

$$(J - X) = ((J_n - X_n), n \geq 0). \tag{1}$$

Supposing $X_0 = 0$, $a.\ s$. The time of state transition is given as $(T_n, n \geq 0)$, where

$$T_n = \sum_{i=1}^{n} X_i. \tag{2}$$

Specially, for continuous time t, the state of system S is represented by r.v. $J(t)$. Let $J(t)$ be the semi-Markov process corresponding to the positive $(J - X)$ process. The time point is recorded as $t_1, t_2, \cdots, t_n$. The state sojourn time is recorded as x_n, $x_n = t_n - t_{n-1}$.

Based on the probability space, $P(J(0) = i) = p_i$, $i = 1, \cdots, m$ with $\sum_{i=1}^{m} p_i = 1$ is given. We assume that for all $n \in N_0, j \in I, t \in R^+$:

$$\begin{aligned} P\left(J\left(t_n\right) = j, x_n \leq t \mid J\left(t_{n-1}\right) = i, x_{n-1}, J\left(t_{n-1}\right), x_{n-2}, \cdots, J_0\right) \\ = P\left(J\left(t_n\right) = j, x_n \leq t \mid J\left(t_{n-1}\right) = i\right). \end{aligned} \tag{3}$$

We noted the running states of system S as

$$Q_{ij}(t) = P\left(J\left(t_n\right) = j, x_n \leq t \mid J\left(t_{n-1}\right) = i\right). \tag{4}$$

where $Q_{ij}(t)$, $i, j \in I$, $t \in [0, t_n - t_{n-1}]$ is a nondecreasing real function null on R^+ such that if

$$p_{ij} = P\left\{J\left(t_n\right) = j, \mid J\left(t_{n-1}\right) = i\right\} = \lim_{t \to \infty} Q_{ij}(t), i, j \in I, \tag{5}$$

then

$$\sum_{j=1}^{m} p_{ij} = 1, i \in I. \tag{6}$$

We can write the matrix: $\mathbf{Q} = [Q_{ij}]$, $\mathbf{P} = [p_{ij}]$.

Definition 2.1. [21] Every $m \times m$ matrix $\mathbf{Q}$ of nondecreasing functions null on R^+ satisfying properties Eqs. 5 and 6 is called a semi-Markov matrix or a semi-Markov kernel.

Definition 2.2. [21] Every couple $(\mathbf{P}, \mathbf{Q})$ where $\mathbf{Q}$ is a semi-Markov kernel and $\mathbf{P}$ a vector of initial probabilities defines a positive $(J - X)$ process as state space and is also called a semi-Markov chain.

The conditional sojourn time distribution function in state i, given that the next state is j, is F_{ij}. The corresponding probability density function is f_{ij}.

We have

$$F_{ij}(t) = P\left(x_n \leqslant t \middle| J\left(t_n\right) = j, J\left(t_{n-1}\right) = i\right) = \int_{-\infty}^{t} f_{ij}(u) = \begin{Bmatrix} \dfrac{Q_{ij}(t)}{p_{ij}} & \text{when } p_{ij} > 0 \\ 1 & \text{when } p_{ij} = 0 \end{Bmatrix} \tag{7}$$

The unconditional distribution function of sojourn time in state i can be denoted as

$$F_i(t) = P\left\{x_n \leqslant t \middle| J\left(t_{n-1}\right) = i\right\} = \sum_{j \in I} Q_{ij}(t) \tag{8}$$

For the calculation of $Q_{ij}(t)$, Reference [22] gives an integral decomposition algorithm, which can be applied to any countable state transitions.

Assuming $F_{ij}(t)$ is known, and the system is currently in j state, it may shift to p state, q state, or o state at next moment. Then we have:

$$\begin{cases} Q_{jp}(t) = \int_0^t \left(1 - F_{jq}(u)\right)\left(1 - F_{jo}(u)\right) dF_{jp}(u) \\ Q_{jq}(t) = \int_0^t \left(1 - F_{jp}(u)\right)\left(1 - F_{jo}(u)\right) dF_{jq}(u) \\ Q_{jo}(t) = \int_0^t \left(1 - F_{jp}(u)\right)\left(1 - F_{jq}(u)\right) dF_{jo}(u) \end{cases}. \tag{9}$$

3 Instantaneous Availability Analysis of Maintenance Process

Support process is a critical link to ensure system availability. The failure modes of an advanced material are unclear, so the supply capacity of spare storage is very weak. In the using process, materials may not get repairs in time after shutting down with a malfunction. Majority commit waits for maintenance; hence, support delay occurs. The three-state situation including work, support delay, and maintenance compared to two-status condition is more corresponding to actuality [23]. There are abundant reasons for support delay such as management, transportation, personnel,

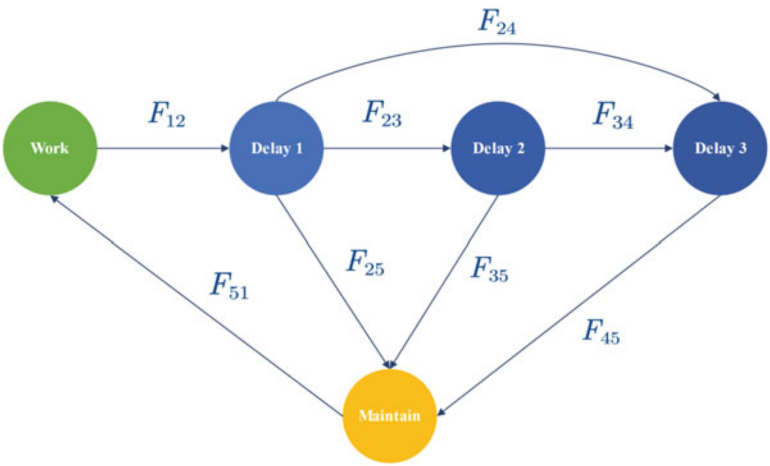

Fig. 1 Sketch map of conditional sojourn time for the semi-Markov process

equipment, spares and storage, etc. We summarized the main factors that cause support delay into three aspects, namely, cover personnel delay time, equipment delay time, and spares delay time.

Based on the above analysis, this section established an instantaneous availability model that considers support delay for repairable system. The possible states of materials are as follows.

State 1: Work. The system is working normally.
State 2: Delay. 1. In the process of support delays, the system is waiting for support personnel to detect after failing.
State 3: Delay. 2. In the process of support delays, the system is waiting for the equipment after failing while support personnel are in place.
State 4: Delay. 3. In the process of support delays, the system is waiting for spares while personnel and equipment are in place.
State 5: Maintain. The system is repairing while support resources are all in place.

Maintenance process of system after failure is shown in Fig. 1.

The support delay process can be regarded as a system with five states. The set of all possible states is noted as $I = \{1, 2, 3, 4, 5\}$.

To keep the system in normal working condition during operating, maintenance and support progress are significant links. The state space $U(U \subseteq I)$ represents normal working state of system where instantaneous availability is the probability of the system that is in the working state:

$$A(t) = P\,(J(t) \in U)\,. \tag{10}$$

We consider the following semi-Markov kernel matrix Q:

$$Q = \begin{bmatrix} 0 & Q_{12} & 0 & 0 & 0 \\ 0 & 0 & Q_{23} & Q_{24} & Q_{25} \\ 0 & 0 & 0 & Q_{34} & Q_{35} \\ 0 & 0 & 0 & 0 & Q_{45} \\ Q_{51} & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

We record that the probability when the system is in the j state at t triggered by the i state is ψ_{ij}:

$$\psi_{ij}(t) = P\left\{J(t) = j | J(0) = i\right\}, \tag{12}$$

which is called trigger probability in the following.

According to the transition process of semi-Markov, solution conclusion is [21]:

$$\psi_{ij}(t) = \delta_{ij}\left(1 - F_i(t)\right) + \sum_{r=1}^{5} \int_0^t \sigma_{ir}(\tau)\, \psi_{rj}(t-\tau)\, d\tau, \tag{13}$$

where

$$\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}, \sigma_{ir}(\tau) = \frac{dQ_{ir}(\tau)}{d\tau}. \tag{14}$$

The instantaneous availability can be calculated as follows:

$$A(t) = \sum_{i \in I} \sum_{j \in U} p_i \psi_{ij}(t). \tag{15}$$

4 Analysis Case of Instantaneous Availability

We study system instantaneous availability in the situation that considers support delay and supposes the system initial state is in working. Hence, the initial probability vector is

$$\boldsymbol{p} = (p_1, p_2, p_3, p_4, p_5) = (1, 0, 0, 0, 0)\,. \tag{16}$$

According to maintenance process experiments of an aeronautic electronic component, the conditional sojourn time distribution can be obtained (Table 1).

According to Eq. 9, solve the element Q_{ij} in the semi-Markov kernel matrix Q:

$$Q_{12}(t) = \int_0^t f_{12}(u)du = F_{12}(t) = 1 - e^{-(t/500)^{1.5}} \tag{17}$$

Table 1 The distribution function of sojourn time in state transition

State i	State j	F_{ij}	Parameter
1	2	Weibull distribution $1 - e^{-(t/\lambda)^k}$	$\lambda = 500$ $k = 1.5$
2	3	Normal distribution $\int_0^t \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(u-\mu)^2}{2\sigma^2}\right) du$	$\mu = 3$ $\sigma^2 = 1$
2	4	Normal distribution $\int_0^t \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(u-\mu)^2}{2\sigma^2}\right) du$	$\mu = 10$ $\sigma^2 = 2$
2	5	Exponential distribution $1 - e^{-\lambda t}$	$\lambda = 0.05$
3	4	Normal distribution $\int_0^t \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(u-\mu)^2}{2\sigma^2}\right) du$	$\mu = 7$ $\sigma^2 = 2$
3	5	Exponential distribution $1 - e^{-\lambda t}$	$\lambda = 0.05$
4	5	Exponential distribution $1 - e^{-\lambda t}$	$\lambda = 0.05$
5	1	Normal distribution $\int_0^t \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(u-\mu)^2}{2\sigma^2}\right) du$	$\mu = 20$ $\sigma^2 = 10$

$$\begin{cases} Q_{23}(t) = \int_0^t (1 - F_{24}(u))\,(1 - F_{25}(u))\,dF_{23}(u) \\ Q_{24}(t) = \int_0^t (1 - F_{23}(u))\,(1 - F_{25}(u))\,dF_{24}(u) \\ Q_{25}(t) = \int_0^t (1 - F_{23}(u))\,(1 - F_{24}(u))\,dF_{25}(u) \end{cases} \tag{18}$$

$$\begin{cases} Q_{34}(t) = \int_0^t (1 - F_{35}(u))\,dF_{34}(u) \\ Q_{35}(t) = \int_0^t (1 - F_{34}(u))\,dF_{35}(u) \end{cases} \tag{19}$$

$$Q_{45}(t) = \int_0^t f_{45}(u)du = F_{45}(t) = 1 - e^{-0.05t} \tag{20}$$

$$Q_{51}(t) = \int_0^t f_{51}(u)du = F_{51}(t) = \int_0^t \frac{1}{\sqrt{200\pi}} \exp\left(\frac{(u-20)^2}{20}\right) du. \tag{21}$$

Because some analytical solutions cannot be obtained, the trapezoidal area method is used to solve the numerical solution as shown in Figs. 2, 3, 4, 5 and 6.

Further, according to Eq. 13, we can get ψ_{ij}. Eq. 13 is a system of 5*5 nonlinear integral equations with convolution operation. We use the special properties of convolution in Fourier transform, which is carried out on both sides of the equation group. Then there are

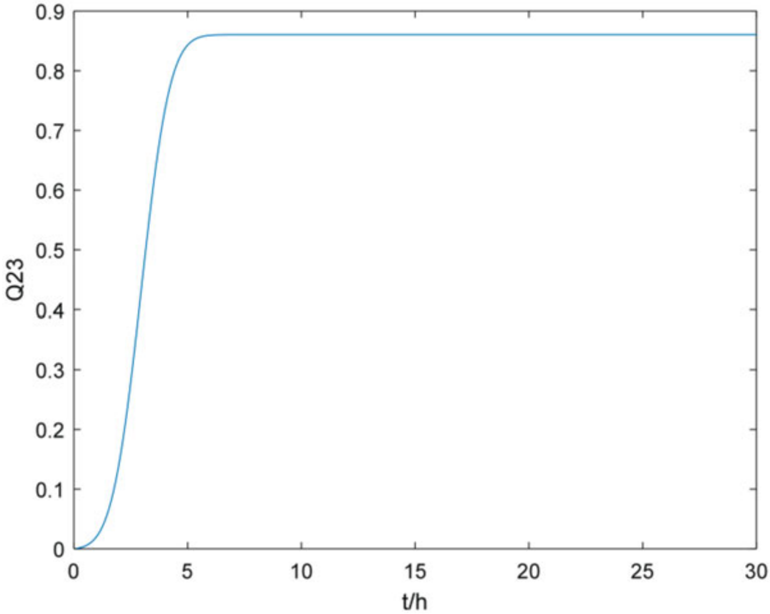

Fig. 2 Fluctuation graph of element Q_{23}

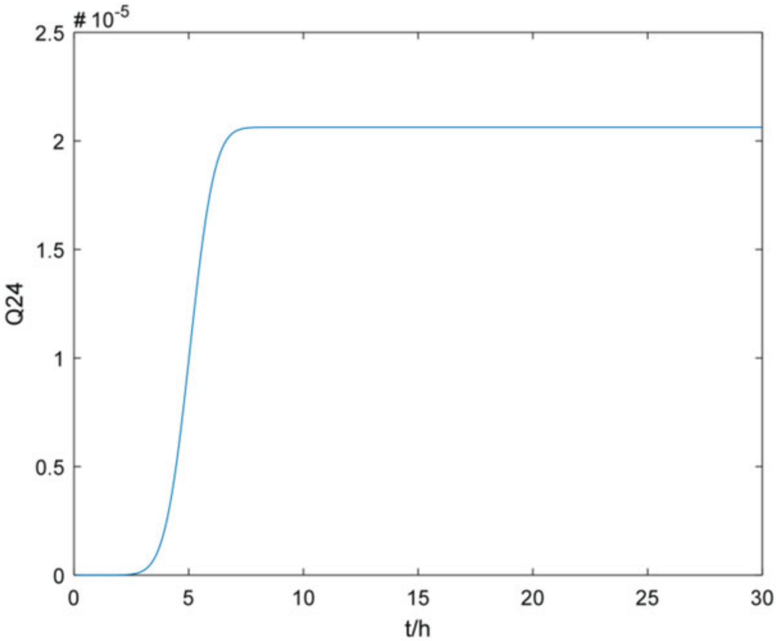

Fig. 3 Fluctuation graph of element Q_{24}

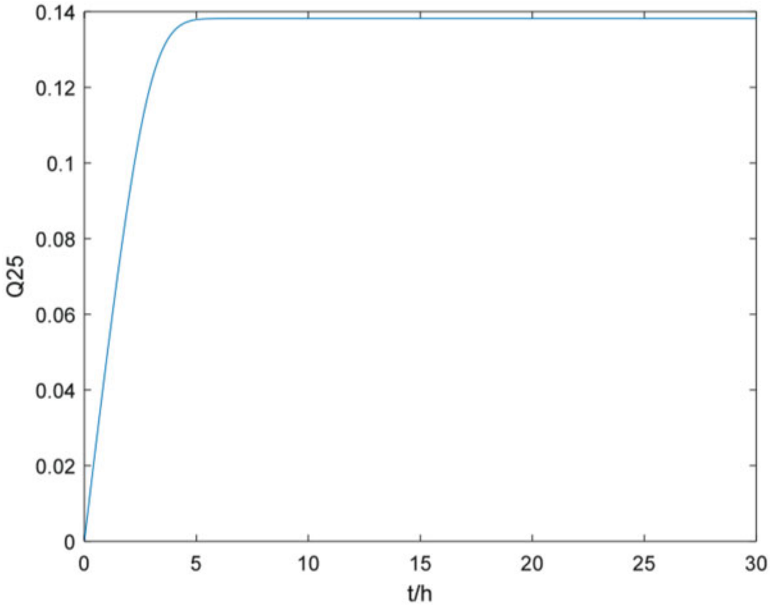

Fig. 4 Fluctuation graph of element Q_{25}

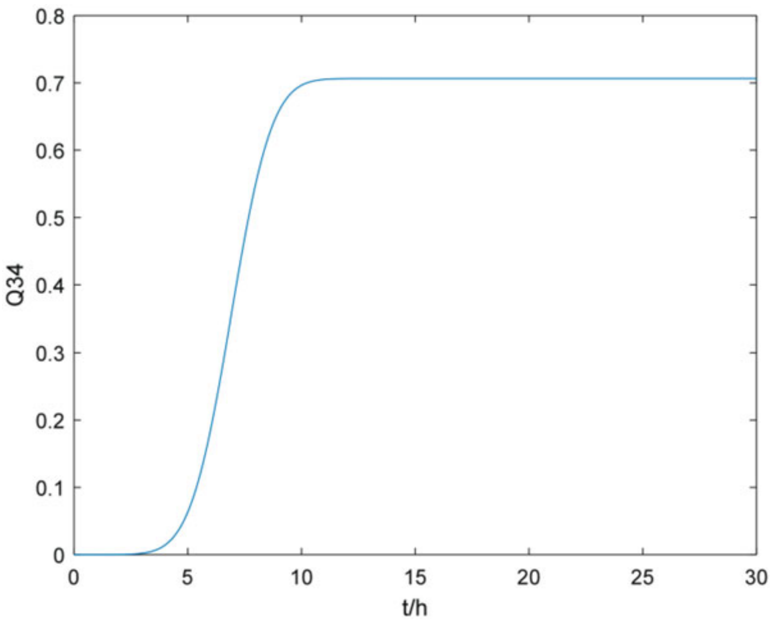

Fig. 5 Fluctuation graph of element Q_{34}

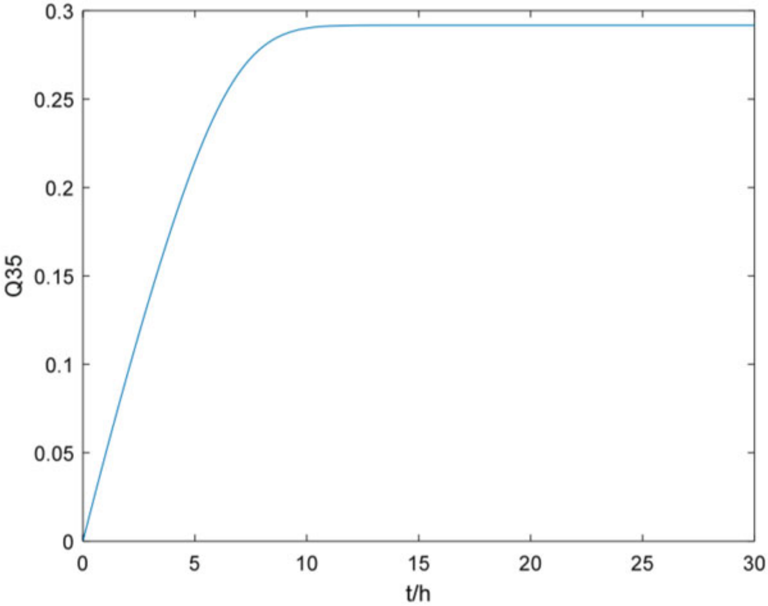

Fig. 6 Fluctuation graph of element Q_{35}

$$\begin{cases} \Upsilon[\psi_{11}] = \Upsilon[1-F_1] + \Upsilon[\sigma_{12}]\Upsilon[\psi_{21}] \\ \Upsilon[\psi_{21}] = \Upsilon[\sigma_{23}]\Upsilon[\psi_{31}] + \Upsilon[\sigma_{24}]\Upsilon[\psi_{41}] + \Upsilon[\sigma_{25}]\Upsilon[\psi_{51}] \\ \Upsilon[\psi_{31}] = \Upsilon[\sigma_{34}]\Upsilon[\psi_{41}] + \Upsilon[\sigma_{35}]\Upsilon[\psi_{51}] \\ \Upsilon[\psi_{41}] = \Upsilon[\sigma_{45}]\Upsilon[\psi_{51}] \\ \Upsilon[\psi_{51}] = \Upsilon[\sigma_{51}]\Upsilon[\psi_{11}] \end{cases}. \tag{22}$$

Υ is the Fourier operator. This change is used to transform the equations into a nonlinear algebraic equations. Finally, the trigger probability can be obtained by using Fourier inverse operator:

$$\psi_{11} = \Upsilon^{-1} \left\{ \frac{\Upsilon[1-F_1]}{1-\Upsilon[\sigma_{12}]\Upsilon[\sigma_{23}]\Upsilon[\sigma_{51}]\cdot(\Upsilon[\sigma_{34}]\Upsilon[\sigma_{45}]+\Upsilon[\sigma_{35}]) - \Upsilon[\sigma_{12}]\Upsilon[\sigma_{51}]\cdot(\Upsilon[\sigma_{24}]\Upsilon[\sigma_{45}]+\Upsilon[\sigma_{25}])} \right\}. \tag{23}$$

Finally, the instantaneous availability can be calculated:

$$A(t) = \sum_{i\in I}\sum_{j\in U} p_i \psi_{ij}(t) = \psi_{11}(t), \tag{24}$$

and its image fluctuating with time is shown in Fig. 7.

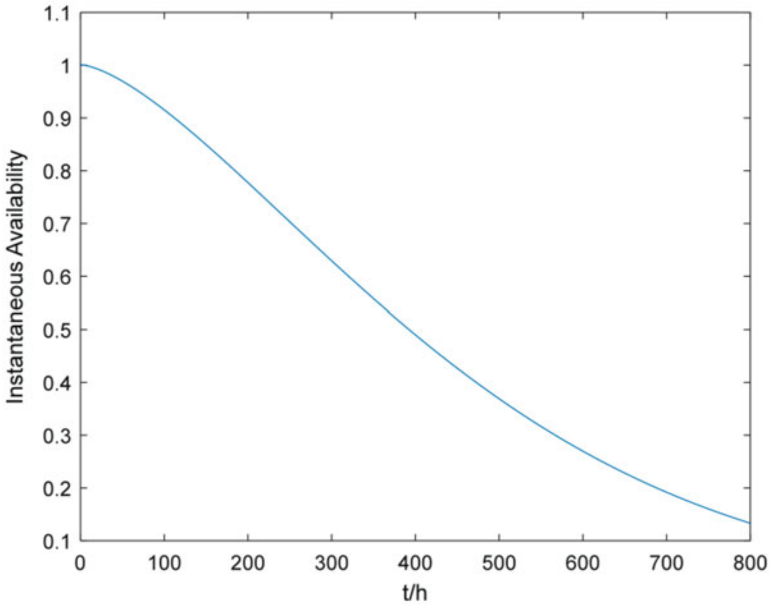

Fig. 7 The curve of instantaneous availability

According to Fig. 7, we can see that the instantaneous availability has an extremely fast drop in early stage, and then it is more stable for a short period of time. With the increase of time, system instantaneous availability decreases gradually. We consider that the system is in the early stage of failure; therefore, the increase of failure rate affects instantaneous availability. Subsequently, system performance is stable, and the availability decreases with time steadily. Finally, in the process of approaching the service life, system enters an unusable state.

5 Conclusion

In this paper, actual situation transforms of system in the process of maintenance and support are considered, and the delay phenomenon of support activities in system is analyzed. The traditional three-state systems with support delay are extended to multi-support factor simultaneous delay system. Based on the semi-Markov process, we first construct support delay model caused by the lack of multiple support resources, which describes the delayed state migration process of support personnel, support equipment, and spares in logistics support. The numerical expression of instantaneous availability with time is obtained by solving convolution equations in Fourier transform. The change of instantaneous availability with time is analyzed, and it is found that the support delay has a great influence on the availability of the system. In order to reduce its impact, we can increase the number of support personnel, improve the attendance rate of support personnel, shorten the time of arrival of support equipment, and reserve sufficient spares. The results enrich the instantaneous availability modeling of multistate system, promote the development of solution technology in semi-Markov transition environment, and implement real-time tracking of instantaneous availability numerical changes. It has considerable engineering value to promote synchronous precision and matching construction of related support resources. In this paper, the maintenance process model corresponds with actuality; therefore, we suppose that it is accessible to prove system maintenance process.

However, the three main influencing factors considered at present are macro. The specific segmentation remains to probe. It is essential to gradually improve maintenance process modeling with various elements that may influence system property. Meanwhile, the influence mechanism of different factors needs to be further explored.

Acknowledgments This work was supported by National Science Foundation of China under Grants 61871013, 71671009 and Consulting Project of China Academy of Engineering:2021XY39. Finally, the authors would like to thank the editors and anonymous reviewers for their insightful comments.

References

1. Yang, Y., Yu, Y.L., Wang, L.C.: Fluctuation Mechanism and Control on System Instantaneous Availability. CRC Press, Boca Raton (2015)
2. Yang, Y., Ren, S.C., Fan, G.M., Kang, R.: Numerical simulation on the existence of fluctuation of instantaneous availability. Trans. Can. Soc. Mech. Eng. **40**(5), 703–713 (2016)
3. Yang, Y., Chen, S.C., Yu, Y.L.: Theoretical analysis of instantaneous availability of systems under uniform distribution. J. Beihang Univ. **42**(001), 28–34 (2016)
4. Yang, J.K., Xu, T.X., Wang, H.W., Mi, Q.L.: Research on the method of modeling availability of missile weapon systems. Tactical Missile Technol. **05**, 30–34 (2012)
5. Wenyuan, L.U., Wang, W., Christer, A.H.: The delay time modeling of preventive maintenance of plant based on subjective PM data and actual failure records. In: International Conference on Quality & Reliability(ICQR2005) School of Management. University of Shanghai for Science & Technology, P O Box475, JunGong Road 516,Shanghai (200093), China (2005)
6. Wang, W.: A two-stage prognosis model in condition based maintenance. Eur. J. Oper. Res. **182**(3), 1177–1187 (2007)
7. Aven, T., Castro, I.T.: A delay-time model with safety constraint. Reliab. Eng. Syst. Saf. **94**(2), 261–267 (2009)
8. Liu, F.S., Wu, W., Shan, Z.W., Chen, Y.: Useability model of armored equipment based on Markov update process. J. Armoured Corps Eng. **24**(005), 19–21 (2010)
9. Wei, Y.Q., Tang, Y.H.: Cold storage and repair system for two different parts based on replacement and repair delay strategy of repair equipment. J. Eng. Math. **37**(04), 423–441 (2020)
10. Yang, Y., Chen, Y., Wen, M.L.: Analysis of instantaneous availability of communication system based on the influence of support equipment. Int. J. Commun. Syst. **31**(99), e3480 (2018)

11. Yongli, Y., Liu, Z.: Basic Theory and Method of Equipment Support Engineering. National Defense Industry Press, Beijing (2015)
12. Lisnianski, A., Frenkel, I., Karagrigoriou, A.: Recent Advances in Multi-State Systems Reliability: Theory and Applications. Springer, Cham (2018)
13. Ruan, Y.P.: Study on Reliability Evaluation Method of Complex System Based on Monte Carlo Simulation. Tianjin University, Tianjin (2013)
14. Li, J.L., Chen, Y.L., Zhang, Y., Huang, H.L.: Availability modeling for periodically inspection system with different lifetime and repair-time distribution. Chin. J. Aeronaut. **32**(7), 1667–1672 (2019)
15. Kong, D.Z., Li, X.B.: A instantaneous availability modelling method for repairable system. Appl. Mech. Mater. **3764**(724), 334–339 (2015)
16. Zhang, H., Meng, D.B., Zong, Y.Y., Wang, F., Xin, T.L.: A modeling and analysis strategy of constellation availability using on-orbit and ground added launch backup and its application in the reliability design for a remote sensing satellite. Adv. Mech. Eng. **10**(4), 1–6 (2018)
17. Fan, R.N.: Theory and Method of Transient Index Approximation in Repairable System. Beijing Institute of Technology, Beijing (2015)
18. Wang, L., Li, M.: Redundancy allocation optimization for multistate systems with failure interactions using semi-Markov process. J. Mech. Des. **137**(10), 101403 (2015)
19. Chryssaphinou, O., Limnios, N., Malefaki, S.: Multi-state reliability systems under discrete time semi-Markovian hypothesis. IEEE Trans. Reliab. **60**(1), 80–87 (2011)
20. Shang, Y.L., Cai, Q., Zhao, X.W., Chen, L.: Multi-state reliability analysis of reactor pump unit based on UGF and semi-Markov methods. Nucl. Power Eng. **33**(1), 117–123 (2012)
21. Janssen, J., Manca, R.: Applied Semi-Markov Processes. Springer, New York (2006)
22. Limnios, N., Opri, G.: An Semi-Markov Processes and Reliability. Birkhauser, Basel (2001)
23. Ren, S., Yang, Y., Chen, Y., Du, Y.: Fluctuation analysis of instantaneous availability under specific distribution. Neurocomputing. **270**, 152–158 (2017)