Optics Letters

Broadband phase noise measurement of single-frequency lasers by the short-fiber recirculating delayed self-heterodyne method

XU CHEN, JINGMIN LIU, JUNJIE JIANG, SHICHAO YANG, AND XIA YU*

School of Instrumentation and Optoelectronic Engineering, Beihang University, Beijing 100191, China
*xiayu@buaa.edu.cn

Received 27 November 2023; revised 21 December 2023; accepted 1 January 2024; posted 4 January 2024; published 25 January 2024

Characterization of single-frequency lasers (SFLs) requires a precise measurement of their phase noise. However, there exists a contradiction between the frequency range and laser phase noise measurement sensitivity in the delay self-heterodyne method. Achieving a broadband and highly sensitive phase noise measurement often requires overlapping the results obtained from different delay lengths. In this study, we present a precisely designed short-fiber recirculating delayed self-heterodyne (SF-RDSH) method that enables the broadband and highly sensitive laser phase noise measurement in a compact setup. By designing the length of the delay fiber based on a theoretical model, the RDSH technique with a shortest delay length of 200 m enables a highly sensitive laser phase noise measurement from 1 Hz to 1 MHz for the first time, to our knowledge. In the experiment, we demonstrate the broadband phase noise measurement of an SFL by analyzing the 1st and 10th beat notes.

https://doi.org/10.1364/OL.514328

Single-frequency lasers (SFLs) are widely used in fields such as high-resolution spectroscopy [1] and optical atomic clocks [2]. Linewidth and power spectral density (PSD) of phase noise are both critical parameters that characterize the properties of an SFL in the frequency domain. Moreover, the PSD of phase noise contains more information than just a linewidth alone, which may reveal important noise characteristics of an SFL. Specifically, laser phase noise in a low-frequency range is a key factor that limits the resolution of optical sensing [3], and the laser phase noise in a high-frequency range presents the quantum noise of the laser. Thus, it is essential to measure the phase noise of an SFL over a wide frequency range to optimize its performance and advance its applications [4].

Previous studies have proposed several methods for measuring the phase noise of SFLs. The first one is based on frequency–intensity conversion through a frequency discriminator, such as the absorption line of the atom and molecules [5] or a reference cavity [6]. To achieve high-resolution measurement, a complex setup is required to lock the frequency of the laser to the resonance with megahertz or even a narrower linewidth. Additionally, this method suffers from inaccuracies resulting from the calibration of the conversion factor. The heterodyne method [7] directly generates the beat notes between the laser under test (LUT) and a reference laser that has similar or negligible phase noise compared to that of the LUT. The laser phase noise can be directly obtained from the RF signal phase noise. However, the heterodyne method requires the central frequency of the beat note to be highly stable, which is hard to achieve during a long measurement time. Furthermore, preparing high-performance reference lasers can also be challenging, especially for lasers operating at certain special wavelengths [8]. The delayed self-heterodyne (DSH) method [9,10], which generates the beat notes through an unbalanced interferometer, has been widely used to measure the phase noise of lasers. Various elaborate DSH methods have been previously proposed, such as a recirculating delayed self-heterodyne [11] and an unbalanced interferometer using a 3×3 coupler [12]. In the DSH methods, the optical fiber delay length L is a critical parameter that determines the frequency range F and the laser phase noise sensitivity factor K of the measurement system [13]. However, there exists a contradiction between the frequency range F and the sensitivity factor K as decreasing the fiber delay length leads to a wider frequency range but less sensitivity to the laser phase noise. Thus, the broadband and highly sensitive laser phase noise measurement often requires employing a variety of phase noise measurement methods or overlapping the results obtained from different delay lengths, but it can complicate the measurement. Therefore, we propose to eliminate this difficulty by using the short-fiber recirculating delayed self-heterodyne (SF-RDSH) method, which can generate beat notes with both long and short delay times simultaneously in a compact setup.

In this paper, we propose a method based on SF-RDSH to sensitively measure the phase noise of an SFL from 1 Hz to 1 MHz within a compact setup. Considering the frequency range and laser phase noise sensitivity, we design the length of the delay fiber to be just 200 m based on a theoretical model. This SF-RDSH with the shortest delay length enables a highly sensitive laser phase noise measurement from 1 Hz to 1 MHz for the first time. We apply the SF-RDSH method to measure the laser phase noise of a high-performance SFL. By analyzing the 1st and 10th beat notes, we measure the phase noise of the laser from 1 Hz to 1 MHz.

In the RDSH method, the phase fluctuations of the RF signal are proportional to the laser phase fluctuations. Considering the intrinsic noise of the measurement system, the relationship

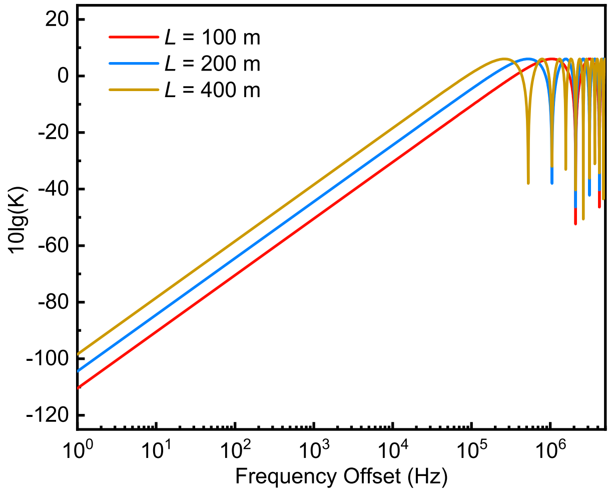

Fig. 1. Simulation of the laser phase noise sensitivity factor K for different delay lengths L of 100, 200, and 400 m.

between the PSD of the laser phase noise $S_\phi(f)$ and the PSD of the RF signal phase noise $S_{\Delta\Phi}(f)$ can be expressed as [11]

$$S_{\Delta\Phi}(f) = 4\sin^2(\pi f \frac{mnL}{c}) S_\Phi(f) + S_{\text{intrinsic}} = K S_\Phi(f) + S_{\text{intrinsic}}, \quad (1)$$

where L is the optical fiber delay length, m is the order of the beat note, n is the refractive index of the optical fiber, c is the speed of the light in a vacuum, and $S_{\text{intrinsic}}$ represents the intrinsic noise from the environmental and detection devices in the system. $K = 4\sin^2\left(\pi f \frac{mnL}{c}\right)$ is defined as the laser phase noise sensitivity factor of the measurement system. Taking the 1st beat note ($m = 1$) for example, the sensitivity factor K has zero points at $F_p = \frac{pc}{nL}$, $p = 1, 2, 3 \ldots$ The first zero point is defined as the upper limit of the frequency range, and the relationship between the fiber length L and the upper limit F can be expressed as

$$F = \frac{c}{nL}. \quad (2)$$

If the measured frequency exceeds the upper limit of the frequency range, multiple oscillations will appear in the results. Figure 1 presents the simulation of sensitivity factor K for delay lengths L of 100, 200, and 400 m in the frequency range of 1 Hz to 5 MHz. As shown in Fig. 1, the sensitivity factor K begins to oscillate at 500 kHz when the delay length L is set to 400 m, but no oscillation occurs when the L is set to 200 m or 100 m at 500 kHz. To obtain the phase noise in a high-frequency range, the delay length L selected should be short. Because components of the laser phase noise in a high-frequency range are basically in the form of white noise, the upper limit of the frequency range of 1 MHz is enough to satisfy the measurement applications. On the other hand, the laser phase noise in a low-frequency range (<100 Hz) is significant for applications, such as strain sensing [3] and gravitational-wave detection [14]. According to Eq. (1), the laser phase noise $S_\phi(f)$ can only be reconstructed from the $S_{\Delta\Phi}(f)$ when we have $KS_\phi(f) \gg S_{\text{intrinsic}}$. However, the value of factor K is relatively low in a low-frequency range when the delay length L is short, resulting in a low signal-to-noise ratio. This poses challenges in detecting the phase noise of ultra-low noise lasers in the low-frequency range. The sensitivity factor K can be enhanced by increasing the order of the beat note in the

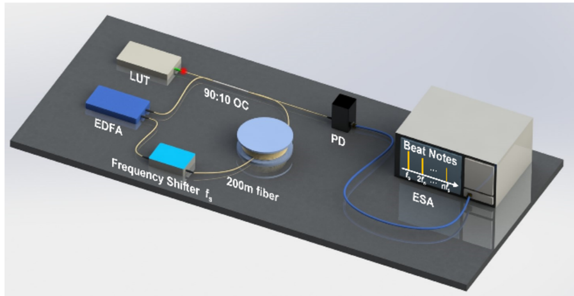

Fig. 2. Measurement setup of the laser phase noise based on SF-RDSH.

SF-RDSH method. In this way, the simultaneous broadband and highly sensitive laser phase noise measurement can be achieved within a compact setup. To achieve the upper frequency limit of 1 MHz, we design the optical delay length L to be 200 m, which is the shortest delay length in the RDSH method. Although the sensitivity factor K increases with higher-order beat notes, it also introduces some spurious components into $S_{\text{intrinsic}}$. Therefore, we select the 10th beat note corresponding to a delay length of 2 km to provide a relatively high sensitivity factor K.

The schematic of the phase noise measurement system based on SF-RDSH is shown in Fig. 2. The coupling ratio of the optical coupler (OC) is optimized as 90:10 based on the RDSH theoretical model [15] to obtain more beat notes in the frequency spectrum. Of the signal beam 90% is injected into the fiber loop while the remaining 10% reference beam enters the photodetector (PD) with a bandwidth of 1.5 GHz. The fiber loop consists of a fiber spool for providing a delay length of 200 m, a frequency shifter, and an Er-doped fiber amplifier (EDFA) for compensating the loss in the fiber loop. The RF frequency of the frequency shifter is 100 MHz, which allows us to obtain 10th beat note within the bandwidth of the PD. The insertion loss in the fiber loop is about 8 dB, and the gain of the EDFA is set to 9 dB to compensate for the loss in the fiber loop to avoid a beat note distortion. The fiber spool is specially designed in a compact format with all fibers stabilized to suppress the external noise from the environment. To further suppress the $S_{\text{intrinsic}}$, we can encapsulate the measurement system into a sealed box with all fibers and devices fixed to isolate the external influence. After m times circulations, the signal beam inside the fiber loop has a frequency shift of $m \cdot 100$ MHz and a delay length of $m \cdot 200$ m. During each circulation, 10% of the signal beam exits the fiber loop and interferes with the reference beam. This interaction generates various beat notes with frequency shifts of $m \cdot 100$ MHz. These beat notes are detected and observed in the electrical signal analyzer (ESA). The RF signal phase noise $S_{\Delta\Phi}(f)$ is measured from those beat notes by the ESA and then converted into laser phase noise $S_\phi(f)$.

In the experiment, a 1550 nm SFL with a linewidth less than 5 kHz is used. The output power of the laser is 10 mW. By analyzing the RF signal at 100 MHz (1st beat note) and 1 GHz (10th beat note), the RF signal phase noise $S_{\Delta\Phi}(f)$ with delay lengths of 200 m and 2 km in the range from 1 Hz to 1 MHz is presented in Fig. 3(a). We note that the lower frequency limit of 1 Hz is determined by the ESA's measurement range. Traditionally, the noise floor of a DSH phase noise measurement system is measured by removing the delay fiber in the setup to eliminate the influence of the laser phase noise [16]. However, because of the pigtails of the optical devices in the fiber loop (~9 m), the influence of

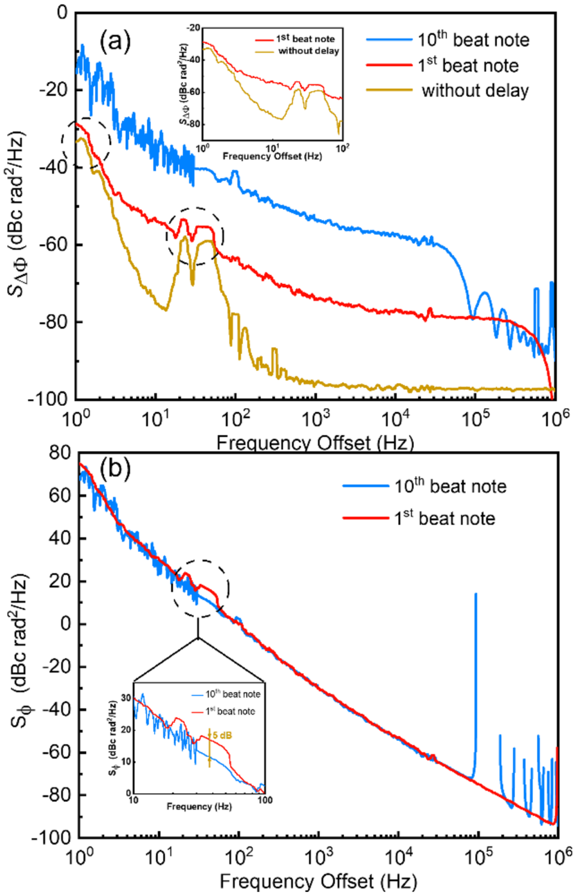

Fig. 3. (a) RF signal phase noise $S_{\Delta\Phi}(f)$ measured from the 1st beat note (read line) and the 10th beat note (blue line). The brown line presents the RF signal phase noise without the delay fiber in the setup. The inset shows the RF signal phase noise $S_{\Delta\Phi}(f)$ from 1 to 100 Hz. (b) Laser phase noise $S_\phi(f)$ measured from the 1st beat note (red line) and the 10th beat note (blue line). The inset shows the laser phase noise $S_\phi(f)$ within the black dashed circle region.

the laser phase noise cannot be totally eliminated. According to Eq. (1), the laser phase noise increases with a longer delay fiber. Therefore, we can still analyze the noise characteristics by measuring the RF signal phase noise without the delay fiber. The RF signal phase noise $S_{\Delta\Phi}(f)$ without the delay fiber is measured and shown in Fig. 3(a). Within the dashed black circle region in Fig. 3(a), the RF signal phase noise without delay closely matches the RF signal phase noise with a delay length of 200 m. The inset shows the details of the RF phase noise from 1 and 100 Hz. This indicates that the $S_{\text{intrinsic}}$ dominates in the frequency range of 1 to 3 Hz and 10 to 100 Hz due to the low phase noise sensitivity factor K. Figure 3(b) presents the calculated laser phase noise from the 1st beat note based on Eq. (1), and there is an abnormal bulge within the black dashed circle region. By circulating the light in the fiber loop to increase the delay length to 2 km and thereby enhancing the sensitivity factor K, the laser phase noise $S_\phi(f)$ becomes dominant, and the influence of $S_{\text{intrinsic}}$ can be neglected. The power of the laser and the gain coefficient of the EDFA are relatively low in the measurement; the additional phase noise from the EDFA mainly results from the amplified spontaneous emission [17]. The calculated the phase noise from the EDFA after multiple circulations is

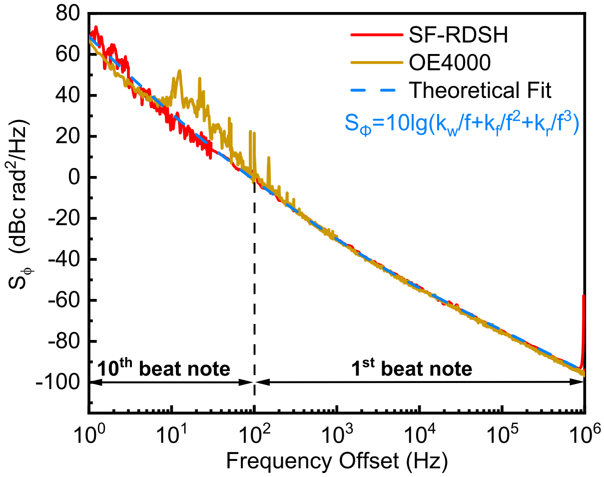

Fig. 4. Laser phase noise measured by SF-RDSH (red line) and OE4000 (brown line). A theoretical laser phase noise (blue dot line) with $k_w = 339$ Hz, $k_f = 6.3 \times 10^5$ Hz^2, and $k_r = 6.7 \times 10^6$ Hz^3 is superimposed.

less than -93 rad^2 dBc/Hz, which is negligible [18]. As shown in the inset of Fig. 3(b), the influence of the $S_{\text{intrinsic}}$ within the black dashed circle region is eliminated by increasing the delay length to 2 km. However, due to the effectively long optical delay length, the laser phase noise with a 2 km delay length begins to exhibit oscillations at 100 kHz.

By combining the measurement results of the 1st and 10th beat notes, we can obtain the laser phase noise $S_\phi(f)$ in the frequency range of 1 Hz to 1 MHz. Specifically, we select the laser phase noise from the 10th beat note for a frequency range of 1 to 100 Hz and the laser phase noise from the 1st beat note for a frequency range of 100 Hz to 1 MHz. The final results are shown in Fig. 4. To validate our measurement results, we compare them with those obtained using OE4000, a widely used commercial instrument for ultra-low laser phase noise measurement [19]. As shown in Fig. 4, the two sets of phase noise results are in good agreement except in the frequency range of 10 to 100 Hz. It should be noted that the spurious components in the OE4000 measurement results may be attributed to the shot noise from the power source or the intensity noise of the laser. It also shows that the by increasing the sensitivity factor K, the influence of external noise can be effectively suppressed, and our measurement system becomes less sensitive to the external noise. Furthermore, we apply a laser phase noise model shown in Fig. 4 to analyze the noise characteristics of the laser phase noise [10], leading to the determination of the white noise coefficient $k_w = 339 \pm 1\%$ Hz, flicker noise coefficient $k_f = 6.3 \times 10^5 \pm 2\%$ Hz^2, and random-walk noise coefficient $k_r = 6.7 \times 10^6 \pm 4\%$ Hz^3. In the frequency range below 10^4 Hz, the flicker noise and random-walk noise are dominated. These noises originate from environmental factors such as fiber vibration, spark noise from the drive circuit, and temperature fluctuations, all of which limit the performance of the SFL. On the other hand, in the frequency range above 10^4 Hz, the laser phase noise takes the form of white noise, which is a result of spontaneous emission.

Based on the laser phase noise $S_\phi(f)$, we can also calculate the frequency noise of the laser and estimate the integral linewidth for different values of the integral bandwidth. As shown in Fig. 5, the integral linewidth of the laser is calculated based on the

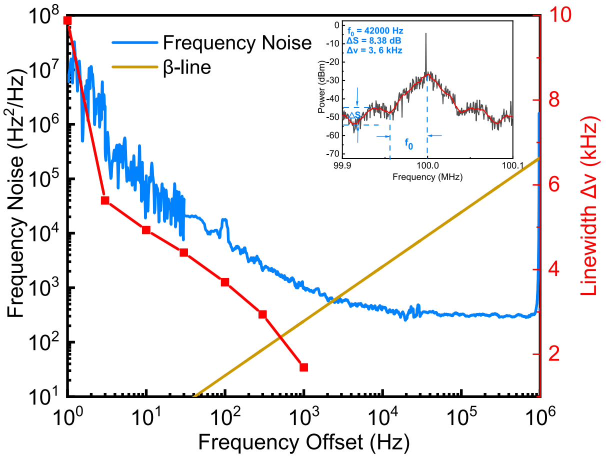

Fig. 5. Frequency noise of the SFL (blue line) and integral linewidth (red line) estimated by the β-line method. The function of the β-line is defined as $S(f) = 8 \ln 2 \times f / \pi^2$. The inset shows the laser linewidth estimated based on the DPA method.

β-line method [20]. The integral linewidth increases as the lower limit of the integration bandwidth decreases because more noise is included in the calculation. At an integral bandwidth of 1 kHz, the minimum linewidth of 1.7 kHz is calculated. In addition, the inset in Fig. 5 shows the linewidth measurement obtained using the dual-parameter acquisition (DPA) method [21]. The beat note is obtained based on the DSH method with a delay length of 5 km. The linewidth calculated is 3.6 kHz. By replacing the delay line with 2 km fiber, the linewidth estimated is 2.7 kHz. Both results based on the DPA method are larger than that estimated from the β-line method due to the presence of the flicker noise [22]. Thus, the linewidth measurement based on the PSD of the frequency noise is recommended because it can eliminate the effect of flicker noise by selecting the appropriate integral bandwidth.

In summary, we, for the first time to the best of our knowledge, have successfully proposed a compact phase noise measurement system utilizing the SF-RDSH method to achieve a broadband and highly sensitive laser phase noise measurement. We design the fiber length in the system to just 200 m based on the theoretical model and apply this method to measure the phase noise of an SFL from 1 Hz to 1 MHz. By combining the analysis of the 1st and 10th beat notes, we measure the laser phase noise across a frequency range from 1 Hz to 1 MHz. We analyze the noise component of the laser phase noise based on the theoretical model and estimate the integral linewidth of the laser based on the β-line method, which helps to comprehensively understand the noise characteristics of the laser. This method offers a promising approach to measure the phase noise of SFLs and advance their applications in the field of high-resolution spectroscopy, strain measurement, and gravitational-wave detection.

Funding. National Natural Science Foundation of China (62175007).

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

REFERENCES

1. D. A. Long, G. W. Truong, R. D. van Zee, *et al.*, Appl. Phys. B **114**, 489 (2014).
2. Z. L. Newman, V. Maurice, T. Drake, *et al.*, Optica **6**, 680 (2019).
3. Q. Zhao, Y. Sun, C. Zeng, *et al.*, Adv. Photonics Res. **3**, 2200080 (2022).
4. F. Lei, Z. Ye, and V. Torres-Company, Opt. Lett. **47**, 513 (2022).
5. X. Chen, J. Chen, J. Luo, *et al.*, IEEE Photonics Technol. Lett. **35**, 834 (2023).
6. F. Schmid, J. Weitenberg, T. W. Hänsch, *et al.*, Opt. Lett. **44**, 2709 (2019).
7. N. Von Bandel, M. Myara, M. Sellahi, *et al.*, Opt. Express **24**, 27961 (2016).
8. W. Ma, B. Xiong, C. Sun, *et al.*, Appl. Opt. **58**, 3555 (2019).
9. T. Okoshi, K. Kikuchi, and A. Nakayama, Electron. Lett. **16**, 630 (1980).
10. S. Camatel and V. Ferrero, J. Lightwave Technol. **26**, 3048 (2008).
11. H. Tsuchida, Opt. Lett. **36**, 681 (2011).
12. D. Xu, F. Yang, D. Chen, *et al.*, Opt. Express **23**, 22386 (2015).
13. O. Llopis, P. H. Merrer, H. Brahimi, *et al.*, Opt. Lett. **36**, 2713 (2011).
14. N. Kenji, W. Y. Anthony, B. C. Jordan, *et al.*, Proc. SPIE **10511**, 105111D (2018).
15. M. Han and A. Wang, Appl. Phys. B **81**, 53 (2005).
16. J. Shi, F. Zhang, and S. Pan, IEEE Trans. Microwave Theory Tech. **66**, 4306 (2018).
17. E. Rochat and R. Dandliker, IEEE J. Sel. Top. Quantum Electron. **7**, 49 (2001).
18. A. W. S. Putra, M. Yamada, H. Tsuda, *et al.*, IEEE J. Quantum Electron. **53**, 5100108 (2017).
19. S. Gundavarapu, G. M. Brodnik, M. Puckett, *et al.*, Nat. Photonics **13**, 60 (2019).
20. G. Di Domenico, S. Schilt, and P. Thomann, Appl. Opt. **49**, 4801 (2010).
21. Z. Wang, C. Ke, Y. Zhong, *et al.*, Opt. Express **28**, 8484 (2020).
22. L. B. Mercer, J. Lightwave Technol. **9**, 485 (1991).