Finite-Time Estimation for Markovian BAM Neural Networks With Asymmetrical Mode-Dependent Delays and Inconstant Measurements

Chang Liu, Zhuo Wang, *Member, IEEE*, Renquan Lu, *Senior Member, IEEE*, Tingwen Huang, *Fellow, IEEE*, and Yong Xu, *Member, IEEE*

***Abstract*— The issue of finite-time state estimation is studied for discrete-time Markovian bidirectional associative memory neural networks. The asymmetrical system mode-dependent (SMD) time-varying delays (TVDs) are considered, which means that the interval of TVDs is SMD. Because the sensors are inevitably influenced by the measurement environments and indirectly influenced by the system mode, a Markov chain, whose transition probability matrix is SMD, is used to describe the inconstant measurement. A nonfragile estimator is designed to improve the robustness of the estimator. The stochastically finite-time bounded stability is guaranteed under certain conditions. Finally, an example is used to clarify the effectiveness of the state estimation.**

***Index Terms*— Finite-time bounded, Markovian bidirectional associative memory neural networks (BAM NNs), state estimation, time-varying delays (TVDs).**

I. Introduction

NEURAL networks (NNs) have a large number of successful applications in various fields [1]–[4], and the research of bidirectional associative memory (BAM) NNs, which was first proposed and researched by Kosko [5], is an important branch. The BAM NNs have important application prospects in automatic control, multifault diagnosis, combination optimization, signal and image processing, and so on [6]. In practice, the structure and parameters of system are not always constant as time goes on, which can be described by the Markovian jump model, and many results about it have been proposed [7]–[12]. In addition, with the widespread application of digital computing units, the study of discrete-time Markovian BAM NNs is particularly important. The time delay is inevitable and usually asymmetrical in the information transmission course between two layers of BAM NNs, which may cause instability. The time delay that occurs in BAM NNs can be classified as constant time delay, time-varying delays (TVDs), distributed delays, and mixed time delays [13]. The stability analysis of BAM NNs with time delays is of great significance, which has been studied and explored by many researchers. Sowmiya *et al.* [14] studied the mean-square asymptotic stability of the impulsive Markovian jumping BAM NNs with distributed TVDs. In [15], the discrete-time BAM NNs with TVDs were explored, and sufficient conditions were obtained to ensure that the augmented system is finite-time bounded. The global robust exponential dissipativity of BAM NNs with mixed time delays was analyzed in [16]. What we have to point out is that the transmission delay always depends on the system operation conditions [17]–[19]. Specially, mode-dependent time delays that are easier to match the actual circumstance were first studied for discrete-time Markovian jump system in [19]. Unfortunately, two decades after Boukas and Liu's work [19], the mode-dependent delays for the discrete-time Markovian jump system have not been fully considered and investigated yet. In practice, for discrete-time Markovian BAM NNs, the delays change in different time intervals for different system modes and different layers, how to model and analyze the delays motivates our further study.

Networked control systems are widely used in the actual systems, where the subsystems are distributed in different places and connected by unreliable networks [20], [21]. In networked control systems, the information measured by the sensors is transmitted to the remote controller or estimator, and the control signals are transmitted back to the systems [22], [23]. The sensors need to locate in the harsh environment to obtain some special measurements [24], where the sensors always meet some challenges, such as scope, energy consumption, and sudden change in temperature. Thus, the derived measurements may be unreliable, and some related works have been derived. In [25], multistep sensor delays and missing

Manuscript received 25 December 2020; revised 14 April 2021; accepted 25 June 2021. Date of publication 16 July 2021; date of current version 5 January 2023. This work was supported in part by the National Natural Science Foundation of China under Grant 61876041, Grant 61875040, and Grant 61903093; in part by the Local Innovative and Research Teams Project of Guangdong Special Support Program under Grant 2019BT02x353; and in part by the Innovative Research Team Program of Guangdong Province Science Foundation under Grant 2018B030312006. *(Corresponding author: Zhuo Wang.)*

Chang Liu, Renquan Lu, and Yong Xu are with the Guangdong Provincial Key Laboratory of Intelligent Decision and Cooperative Control, School of Automation, Guangdong University of Technology, Guangzhou 510006, China (e-mail: liuchangwm@163.com; rqlu@gdut.edu.cn; xuyong809@163.com).

Zhuo Wang is with the Research Institute of Frontier Science, Beihang University, Beijing 100191, China, also with the Key Laboratory of Ministry of Industry and Information Technology on Quantum Sensing Technology, Beihang University, Beijing 100191, China, also with the Beijing Advanced Innovation Center for Big Data-Based Precision Medicine, Beihang University, Beijing 100191, China, and also with the Beijing Academy of Quantum Information Sciences, Beijing 100193, China (e-mail: zhuowang@buaa.edu.cn).

Tingwen Huang is with the Department of Science Program, Texas A&M University at Qatar, Doha, Qatar (e-mail: tingwen.huang@qatar.tamu.edu).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TNNLS.2021.3094551.

Digital Object Identifier 10.1109/TNNLS.2021.3094551

measurements were considered, which were caused by noisy environment and unreliable communication. Yan et al. [26] researched multisensor systems with heavy-tailed noises and unreliable measurements, where different sensors were likely to work with different sampling rates. Therefore, it is an important topic that further investigates the characteristics of sensors and the real situations to derive a more general measurement model.

In most cases, the sensors cannot obtain the state directly, and thus, how to design an estimator to estimate the system state by partially observable information is a research hotspot and attracts a lot of attention [27]–[32]. In [33], for the discrete-time Markov jump NNs, an asynchronous and resilient filter was proposed. In [34], a buffer-dependent smart estimator was designed for networked systems with a Markov-driven transmission strategy, and the buffer with limited capacity was considered. For the channel fading and dynamic event-triggered strategy, the nonfragile estimator is able to improve the robustness of the estimator, where the uncertainty is within permitted interval [35]. By searching a mass of relevant references, the nonfragile estimator design using the polytopic uncertainty (PU) model has not been fully considered.

The classical control theory mainly deals with the asymptotic behavior of the system trajectory in an infinite interval. In practice, most of the research works focus on dynamic performance over a finite time, such as the problem of controlling and tracking for autonomous underwater vehicles from an initial point to a final point in a prescribed time interval [36]. Some latest related results about finite-time boundedness, finite-time stability, and finite-time state estimation can be found in [37]–[40]. However, to the best of our knowledge, the finite-time state estimation problem for discrete-time Markovian BAM NNs with asymmetrical system mode-dependent (SMD) TVDs and inconstant measurements has not been fully analyzed yet and needs to be further investigated.

This work studies the finite-time state estimation for Markovian BAM NNs with asymmetrical SMD TVDs and inconstant measurements. A nonfragile estimator is designed by the PU analysis method to ensure that the estimation error system (EES) meets the stochastically finite-time bounded (SFTB). An example is used to clarify the results. The main contributions are as follows.

1) A novel model of asymmetrical SMD TVDs is considered for discrete-time Markovian BAM NNs, where the TVDs change in different time intervals for different system modes and different layers. Compared with the existing ones [14]–[19], the SMD TVDs pay more attention to the change interval of TVDs, which are more detailed and match the actual circumstance.
2) A general Markov chain $\vartheta(k) \in S = \{1, 2, \ldots, s\}$ is used to describe the conditions of the measurement, where the transition probability matrix is SMD. Furthermore, a new Lyapunov–Krasovskii function is established to analyze the Markovian BAM NNs with SMD TVDs and inconstant measurements.

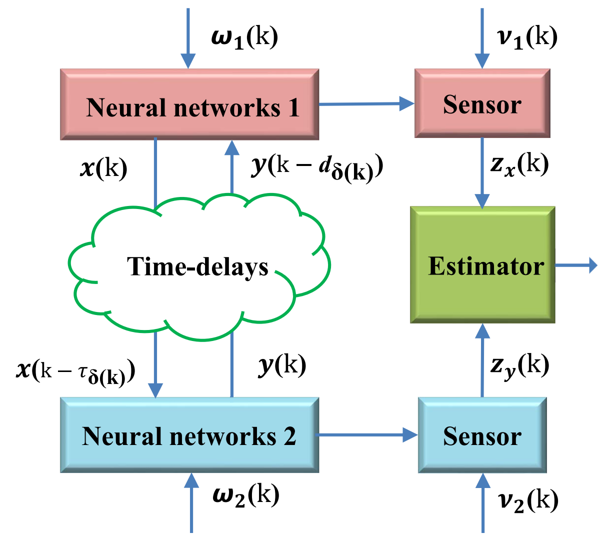

Fig. 1. Structure diagram of state estimation for Markovian BAM NNs with asymmetrical SMD TVDs and inconstant measurements.

3) The PU model is employed to describe the uncertainty of the estimator gains, which is less conservative than the interval uncertainty (IU) model [35]. In the end, the simulation results depend on selection of design parameters, and their effects on performance of the proposed method are analyzed through simulation experiments.

The organization of this article is as follows. In Section II, the discrete Markovian BAM NNs with asymmetrical SMD TVDs and inconstant measurements are described, and a nonfragile estimator is designed. Sufficient conditions of the SFTB are given, and the nonfragile estimator gains are obtained in Section III. In Section IV, a numerical simulation is given, and conclusions are given in the end.

Notations: The symbols $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional vectors and $m \times n$ real matrices, respectively. The symbols $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ are, respectively, denoted as the smallest eigenvalue and largest eigenvalue for matrix X. The matrix X^T is the transpose of matrix X, and the symbol diag$\{\cdot\}$ stands for the diagonal matrix. The symbol $\mathbb{E}\{\cdot\}$ means the expectation of a stochastic variable.

II. System Description and Preliminaries

A. Markovian BAM NNs Description

As shown in Fig. 1, the discrete-time Markovian BAM NNs with asymmetrical SMD TVDs are considered as

$$\begin{cases} \boldsymbol{x}(k+1) = A_{\delta(k)}\boldsymbol{x}(k) + B_{\delta(k)}\boldsymbol{g}(\boldsymbol{y}(k-d_{\delta(k)}(k))) \\ \qquad\qquad + E_{\delta(k)}\boldsymbol{\omega}_1(k) \\ \boldsymbol{y}(k+1) = C_{\delta(k)}\boldsymbol{y}(k) + D_{\delta(k)}\boldsymbol{g}(\boldsymbol{x}(k-\tau_{\delta(k)}(k))) \\ \qquad\qquad + F_{\delta(k)}\boldsymbol{\omega}_2(k) \\ \boldsymbol{x}(\iota_1) = \phi_1(\iota_1), \quad \iota_1 \in \{-\tau_M, -\tau_M+1, \ldots, 0\} \\ \boldsymbol{y}(\iota_2) = \phi_2(\iota_2), \quad \iota_2 \in \{-d_M, -d_M+1, \ldots, 0\} \end{cases} \tag{1}$$

where $\boldsymbol{x}(k) \in \mathbb{R}^n$ and $\boldsymbol{y}(k) \in \mathbb{R}^n$ are state vectors and $A_{\delta(k)} \in \mathbb{R}^{n\times n}$, $B_{\delta(k)} \in \mathbb{R}^{n\times n}$, $C_{\delta(k)} \in \mathbb{R}^{n\times n}$, $D_{\delta(k)} \in \mathbb{R}^{n\times n}$, $E_{\delta(k)} \in \mathbb{R}^{n\times p_1}$, and $F_{\delta(k)} \in \mathbb{R}^{n\times p_2}$ are known matrices.

The stochastic variable $\delta(k)$ is a Markov chain, and its states $\delta(k) \in \mathfrak{S} = \{1, 2, \dots, \mathfrak{s}\}$ obey the transition probability

$$\pi_{qr} = \mathbb{P}\{\delta(k+1) = r|\delta(k) = q\}, \quad q, r \in \mathfrak{S}. \tag{2}$$

The transition probability matrix of the Markov chain is $\Pi_\delta = [\,\pi_{qr}\,]_{\mathfrak{s}\times\mathfrak{s}}$ with $\sum_{r=1}^{\mathfrak{s}} \pi_{qr} = 1,\ \forall q \in \mathfrak{S}$.

The scalars $d_{\delta(k)}(k)$ and $\tau_{\delta(k)}(k)$ represent the asymmetrical SMD integer TVDs. The time delays vary in different time intervals under different system modes, that is, the integer TVDs satisfy the following inequalities:

$$\begin{aligned} d_{m\delta(k)} &\le d_{\delta(k)}(k) \le d_{M\delta(k)} \\ \tau_{m\delta(k)} &\le \tau_{\delta(k)}(k) \le \tau_{M\delta(k)} \end{aligned} \tag{3}$$

where $d_{M\delta(k)}$, $d_{m\delta(k)}$, $\tau_{M\delta(k)}$, and $\tau_{m\delta(k)}$ are, respectively, represent the upper and lower bound of the asymmetrical TVDs $d_{\delta(k)}(k)$ and $\tau_{\delta(k)}(k)$. The scalars $\tau_M \triangleq \max_{\delta(k)\in\mathfrak{S}}\{\tau_{M\delta(k)}\}$, $\tau_m \triangleq \min_{\delta(k)\in\mathfrak{S}}\{\tau_{m\delta(k)}\}$, $d_M \triangleq \max_{\delta(k)\in\mathfrak{S}}\{d_{M\delta(k)}\}$, and $d_m \triangleq \min_{\delta(k)\in\mathfrak{S}}\{d_{m\delta(k)}\}$ are known parameters.

The vectors $\boldsymbol{\omega}_1(k) \in \mathbb{R}^{p_1}$ and $\boldsymbol{\omega}_2(k) \in \mathbb{R}^{p_2}$ are noise sequences in a finite horizon $[0, N]$ and satisfy

$$\sum_{k=0}^{N} (\boldsymbol{\omega}_1(k)^T \boldsymbol{\omega}_1(k) + \boldsymbol{\omega}_2(k)^T \boldsymbol{\omega}_2(k)) < c_1^2. \tag{4}$$

The nonlinear neuron activation function $\boldsymbol{g}(\boldsymbol{x}(k))$ of the Markovian BAM NNs is assumed to satisfy the following condition.

Assumption 1: The nondecreasing nonlinear function $\boldsymbol{g}_i(\cdot)$, $i \in \{1, 2, \dots, n\}$, is continuous, and there exists a constant $\bar{\boldsymbol{\ell}}_i$ such that the inequality (5) holds [41]

$$0 \le \frac{\boldsymbol{g}_i(a) - \boldsymbol{g}_i(b)}{a-b} \le \bar{\boldsymbol{\ell}}_i \tag{5}$$

where $a, b \in \mathbb{R}$ and $a \neq b$.

In the actual condition, the sensors are inevitably influenced by the environment disturbances and the system operation condition. Thus, the measurements of the Markovian BAM NNs with asymmetrical SMD TVDs are

$$\begin{cases} \boldsymbol{z}_x(k) = L_{x,\vartheta(k)}\boldsymbol{x}(k) + M_{x,\vartheta(k)}\boldsymbol{v}_1(k) \\ \boldsymbol{z}_y(k) = L_{y,\vartheta(k)}\boldsymbol{y}(k) + M_{y,\vartheta(k)}\boldsymbol{v}_2(k) \end{cases} \tag{6}$$

where $\boldsymbol{z}_x(k) \in \mathbb{R}^m$ and $\boldsymbol{z}_y(k) \in \mathbb{R}^m$ are the measurement vectors, $L_{x,\vartheta(k)} \in \mathbb{R}^{m\times n}$, $L_{y,\vartheta(k)} \in \mathbb{R}^{m\times n}$, $M_{x,\vartheta(k)} \in \mathbb{R}^{m\times q_1}$, and $M_{y,\vartheta(k)} \in \mathbb{R}^{m\times q_2}$ are known matrices. $\vartheta(k) \in S = \{1, 2, \dots, s\}$ is a Markov chain with the transition probability matrices $\Pi_\vartheta^{\delta(k)} = \big[\pi_{\imath\jmath}^{\delta(k)}\big]_{s\times s}$ given by

$$\pi_{\imath\jmath}^{\delta(k)} = \mathbb{P}\{\vartheta(k+1) = \jmath|\vartheta(k) = \imath, \delta(k)\} \tag{7}$$

where $0 \le \pi_{\imath\jmath}^{\delta(k)} \le 1$ for all $\imath, \jmath \in S$, $\delta(k) \in \mathfrak{S}$, and $\sum_{\jmath=1}^{s} \pi_{\imath\jmath}^{\delta(k)} = 1$ for all $\imath \in S$, $\delta(k) \in \mathfrak{S}$. $\delta(k)$ is a Markov chain and the mode is available at instant k.

The noise vectors $\boldsymbol{v}_1(k) \in \mathbb{R}^{q_1}$ and $\boldsymbol{v}_2(k) \in \mathbb{R}^{q_2}$ satisfy the following condition:

$$\sum_{k=0}^{N} (\boldsymbol{v}_1(k)^T \boldsymbol{v}_1(k) + \boldsymbol{v}_2(k)^T \boldsymbol{v}_2(k)) < c_2^2. \tag{8}$$

Remark 1: The measurements obtained by the sensors are always unideal. There are two reasons that influence the measurements. On the one hand, the environment of the system is inconstant, including temperature and humidity. On the other hand, the measurements are also related to the system operating condition. Thus, a Markov chain $\vartheta(k)$ whose transition probability depends on $\delta(k)$ is used to describe the measurement condition.

B. State Estimator and EES

For the reason that there exists an uncertainty of the estimator, the following nonfragile state estimator is designed to increase the reliability of the estimator:

$$\begin{cases} \hat{\boldsymbol{x}}(k+1) = A_{\delta(k)}\hat{\boldsymbol{x}}(k) + B_{\delta(k)}\boldsymbol{g}(\hat{\boldsymbol{y}}(k-d_{\delta(k)}(k))) \\ \qquad\qquad + K_{x,\alpha}(\boldsymbol{z}_x(k) - L_{x,\vartheta(k)}\hat{\boldsymbol{x}}(k)) \\ \hat{\boldsymbol{y}}(k+1) = C_{\delta(k)}\hat{\boldsymbol{y}}(k) + D_{\delta(k)}\boldsymbol{g}(\hat{\boldsymbol{x}}(k-\tau_{\delta(k)}(k))) \\ \qquad\qquad + K_{y,\beta}(\boldsymbol{z}_y(k) - L_{y,\vartheta(k)}\hat{\boldsymbol{y}}(k)) \\ \hat{\boldsymbol{z}}_x(k) = L_{x,\vartheta(k)}\hat{\boldsymbol{x}}(k) \\ \hat{\boldsymbol{z}}_y(k) = L_{y,\vartheta(k)}\hat{\boldsymbol{y}}(k) \\ \hat{\boldsymbol{x}}(\imath) = \hat{\boldsymbol{y}}(\imath) = 0 \end{cases} \tag{9}$$

where $\imath \in \{-\chi_M, -\chi_M + 1, \dots, 0\}$, $\chi_M = \max\{\tau_M, d_M\}$. The state vector $\hat{\boldsymbol{x}}(k) \in \mathbb{R}^n$ (or $\hat{\boldsymbol{y}}(k) \in \mathbb{R}^n$) is the estimation of the state $\boldsymbol{x}(k)$ (or $\boldsymbol{y}(k)$), and the output of the estimator is $\hat{\boldsymbol{z}}_x(k) \in \mathbb{R}^m$ (or $\hat{\boldsymbol{z}}_y(k) \in \mathbb{R}^m$). The matrices $K_{x,\alpha} \in \mathbb{R}^{n\times m}$ and $K_{y,\beta} \in \mathbb{R}^{n\times m}$ are the estimator gains that will be designed later. For the purpose to improve the robustness of the estimator, the nonfragile estimator gains are considered, which are described by the following PU model:

$$\begin{aligned} K_{x,\alpha} &\triangleq \alpha K_{x,1} + (1-\alpha)K_{x,2}, \quad 0 \leqslant \alpha \leqslant 1 \\ K_{y,\beta} &\triangleq \beta K_{y,1} + (1-\beta)K_{y,2}, \quad 0 \leqslant \beta \leqslant 1 \end{aligned} \tag{10}$$

where matrices $K_{x,1}$, $K_{x,2}$, $K_{y,1}$, and $K_{y,2}$ stand for the vertices of the polytopes and satisfy the equalities (11) and the scalars α and β are time-invariants

$$\begin{aligned} K_{x,2} - K_{x,1} &= \urcorner \\ K_{y,2} - K_{y,1} &= \Gamma \end{aligned} \tag{11}$$

where $\urcorner$ and Γ are known matrices.

Define $\boldsymbol{e}_x(k) \triangleq \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$ and $\boldsymbol{e}_y(k) \triangleq \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)$ as the state estimation errors, and $\boldsymbol{e}_{z_x}(k) \triangleq \boldsymbol{z}_x(k) - \hat{\boldsymbol{z}}_x(k)$ and $\boldsymbol{e}_{z_y}(k) \triangleq \boldsymbol{z}_y(k) - \hat{\boldsymbol{z}}_y(k)$ are the measurement estimation errors, also denote $\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_{\delta(k)}(k))) \triangleq \boldsymbol{g}(\boldsymbol{y}(k-d_{\delta(k)}(k))) - \boldsymbol{g}(\hat{\boldsymbol{y}}(k-d_{\delta(k)}(k)))$, $\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_{\delta(k)}(k))) \triangleq \boldsymbol{g}(\boldsymbol{x}(k-\tau_{\delta(k)}(k))) - \boldsymbol{g}(\hat{\boldsymbol{x}}(k-\tau_{\delta(k)}(k)))$, $\boldsymbol{v}_1(k) \triangleq [\boldsymbol{\omega}_1(k)^T\ \ \boldsymbol{v}_1(k)^T]^T$, and $\boldsymbol{v}_2(k) \triangleq [\boldsymbol{\omega}_2(k)^T\ \ \boldsymbol{v}_2(k)^T]^T$. Then, the EES of the Markovian BAM NNs can be obtained from (1) and (9)

$$\begin{cases} \boldsymbol{e}_x(k+1) = \bar{A}_{\delta(k),\vartheta(k)}\boldsymbol{e}_x(k) + \bar{E}_{\delta(k),\vartheta(k)}\boldsymbol{v}_1(k) \\ \qquad\qquad + B_{\delta(k)}\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_{\delta(k)}(k))) \\ \boldsymbol{e}_y(k+1) = \bar{C}_{\delta(k),\vartheta(k)}\boldsymbol{e}_y(k) + \bar{F}_{\delta(k),\vartheta(k)}\boldsymbol{v}_2(k) \\ \qquad\qquad + D_{\delta(k)}\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_{\delta(k)}(k))) \\ \boldsymbol{e}_x(\imath_1) = \phi_1(\imath_1),\ \ \boldsymbol{e}_y(\imath_1) = \phi_2(\imath_2) \end{cases} \tag{12}$$

where

$$\begin{aligned}\bar{E}_{\delta(k),\vartheta(k)} &= [E_{\delta(k)} - K_{x,\alpha}M_{x,\vartheta(k)}]\\ \bar{F}_{\delta(k),\vartheta(k)} &= [F_{\delta(k)} - K_{y,\beta}M_{y,\vartheta(k)}]\\ \bar{A}_{\delta(k),\vartheta(k)} &= A_{\delta(k)} - K_{x,\alpha}L_{x,\vartheta(k)}\\ \bar{C}_{\delta(k),\vartheta(k)} &= C_{\delta(k)} - K_{y,\beta}L_{y,\vartheta(k)}.\end{aligned}$$

Definition 1: The EES (12) satisfies the SFTB, if there exist positive scalars δ_1 and δ_2 ($\delta_2 \geq \delta_1 > 0$), a positive definite matrix R satisfying that the inequalities (4), (8), and (13) hold, for $\iota \in \{-\chi_M, -\chi_M+1, \ldots, 0\}$ [41]

$$\mathbb{E}\{\boldsymbol{e}_x(\iota)^T R\boldsymbol{e}_x(\iota) + \boldsymbol{e}_y(\iota)^T R\boldsymbol{e}_y(\iota)\} \leq \delta_1^2. \tag{13}$$

Then, the following inequality holds for $\forall k \in \{1, 2, \ldots, N\}$:

$$\mathbb{E}\{\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k) + \boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k)\} \leq \delta_2^2. \tag{14}$$

III. Main Results

This section is devoted to studying the sufficient conditions that the EES (12) satisfies the SFTB. Before the presentation, define

$$\begin{aligned}\boldsymbol{\ell}_1 &\triangleq \mathrm{diag}\{\bar{\boldsymbol{\ell}}_1, \bar{\boldsymbol{\ell}}_2, \ldots, \bar{\boldsymbol{\ell}}_n\}\\ \boldsymbol{\ell}_2 &\triangleq \frac{1}{2}\boldsymbol{\ell}_1,\ \ \bar{R} = R^{-\frac{1}{2}}\boldsymbol{\ell}_1^T\boldsymbol{\ell}_1 R^{-\frac{1}{2}}.\end{aligned}$$

Theorem 1: Given a scalar $\gamma > 1$, the EES (12) is SFTB, if there exist matrices $P_{q,\iota} \succ 0$, $Q_{q,\iota} \succ 0$, $P_2 \succ 0$, $P_3 \succ 0$, $Q_2 \succ 0$, $Q_3 \succ 0$, $\mathcal{S}_1 \succ 0$, $\mathcal{S}_2 \succ 0$, $\boldsymbol{\epsilon}_1 \succ 0$, $\boldsymbol{\epsilon}_2 \succ 0$, $\boldsymbol{\varepsilon}_1 \succ 0$, $\boldsymbol{\varepsilon}_2 \succ 0$, $K_{x,1}$ and $K_{y,1}$, and a scalar $l_0 > 0$, such that the following matrix inequalities hold for $\forall q \in \mathfrak{S}$, $\iota \in S$:

$$\begin{bmatrix}\Psi_{11} & \Psi_{12} & 0 & 0 & \Psi_{15} & 0\\ * & \Psi_{22} & 0 & 0 & 0 & \Psi_{26}\\ * & * & -\mathcal{S}_1 & 0 & \bar{E}_{q,\iota}^T & 0\\ * & * & * & -\mathcal{S}_2 & 0 & \bar{F}_{q,\iota}^T\\ * & * & * & * & -\bar{P}_{r,\jmath}^{-1} & 0\\ * & * & * & * & * & -\bar{Q}_{r,\jmath}^{-1}\end{bmatrix} \prec 0 \tag{15}$$

$$\min_{\delta(0)\in\mathfrak{S},\vartheta(0)\in S}\{\lambda_{\min}(\tilde{P}_{\delta(0),\vartheta(0)})\} > l_0 \tag{16}$$

$$\min_{\delta(0)\in\mathfrak{S},\vartheta(0)\in S}\{\lambda_{\min}(\tilde{Q}_{\delta(0),\vartheta(0)})\} > l_0 \tag{17}$$

$$\gamma^N\left(\max_{\delta(0)\in\mathfrak{S},\vartheta(0)\in S}\{\lambda_{\delta(0),\vartheta(0)}\}\delta_1^2 + \lambda_{\max}(\mathcal{S})c^2\right) \leq l_0\delta_2^2 \tag{18}$$

where

$$\begin{aligned}\Psi_{11} &= \mathrm{diag}\{\Psi_{111}, \Psi_{112}, \Psi_{113}, \Psi_{114}\}\\ \Psi_{12} &= \begin{bmatrix}0 & 0 & \boldsymbol{\epsilon}_1\boldsymbol{\ell}_2 & 0\\ 0 & 0 & 0 & \boldsymbol{\epsilon}_2\boldsymbol{\ell}_2\\ \boldsymbol{\ell}_2^T\boldsymbol{\varepsilon}_1^T & 0 & 0 & 0\\ 0 & \boldsymbol{\ell}_2^T\boldsymbol{\varepsilon}_2^T & 0 & 0\end{bmatrix}\\ \Psi_{111} &= -\gamma P_{q,\iota} + \bar{\pi}_{\tau,q}P_2 + \pi_{q,q}(\tau_{Mq} - \tau_{mq})P_2\\ \Psi_{112} &= -\gamma^{\tau_{mq}}P_2, \quad \bar{\pi}_{\tau,q} = \bar{\pi}_q(\tau_M - \tau_m) + 1\\ \Psi_{113} &= (d_{Mq} - d_{mq} + 1)Q_3 - \boldsymbol{\varepsilon}_1\\ \Psi_{114} &= -\gamma^{d_{mq}}Q_3 - \boldsymbol{\varepsilon}_2, \quad \bar{\pi}_q = 1 - \pi_{q,q}\\ \Psi_{22} &= \mathrm{diag}\{\Psi_{221}, \Psi_{222}, \Psi_{223}, \Psi_{224}\}\\ \Psi_{221} &= -\gamma Q_{q,\iota} + \bar{\pi}_{d,q}Q_2 + \pi_{q,q}(d_{Mq} - d_{mq})Q_2\\ \Psi_{222} &= -\gamma^{d_{mq}}Q_2, \quad \bar{\pi}_{d,q} = \bar{\pi}_q(d_M - d_m) + 1\\ \Psi_{223} &= (\tau_{Mq} - \tau_{mq} + 1)P_3 - \boldsymbol{\epsilon}_1\\ \Psi_{224} &= -\gamma^{\tau_{mq}}P_3 - \boldsymbol{\epsilon}_2, \quad c^2 = c_1^2 + c_2^2\\ \Psi_{15} &= \begin{bmatrix}\bar{A}_{q,\iota}^T\\ 0\\ 0\\ B_q^T\end{bmatrix}, \quad \Psi_{26} = \begin{bmatrix}\bar{C}_{q,\iota}^T\\ 0\\ 0\\ D_q^T\end{bmatrix}\\ \bar{P}_{r,\jmath} &= \sum_{\jmath=1}^{s}\sum_{r=1}^{\mathfrak{s}}\pi_{\iota\jmath}^q\pi_{qr}P_{r,\jmath}, \quad \bar{Q}_{r,\jmath} = \sum_{\jmath=1}^{s}\sum_{r=1}^{\mathfrak{s}}\pi_{\iota\jmath}^q\pi_{qr}Q_{r,\jmath}\end{aligned}$$

$$\begin{aligned}\tilde{P}_{\delta(0),\vartheta(0)} &= R^{-\frac{1}{2}}P_{\delta(0),\vartheta(0)}R^{-\frac{1}{2}},\ \ \tilde{P}_2 = R^{-\frac{1}{2}}P_2R^{-\frac{1}{2}}\\ \tilde{Q}_{\delta(0),\vartheta(0)} &= R^{-\frac{1}{2}}Q_{\delta(0),\vartheta(0)}R^{-\frac{1}{2}},\ \ \tilde{Q}_2 = R^{-\frac{1}{2}}Q_2R^{-\frac{1}{2}}\\ \lambda_{\delta(0),\vartheta(0)} &= \lambda_{\max}(\tilde{P}_{\delta(0),\vartheta(0)}) + \lambda_{\max}(\tilde{Q}_{\delta(0),\vartheta(0)})\\ &\quad + \lambda_{\max}(\tilde{P}_2)\big(1 + \bar{\pi}_{\delta(0)}(\tau_M - \tau_m)\\ &\quad + \pi_{\delta(0),\delta(0)}(\tau_{M\delta(0)} - \tau_{m\delta(0)}) - \gamma^{\tau_{M\delta(0)}}\\ &\quad + (\bar{\pi}_{\delta(0)}(\gamma^{\tau_M} - \gamma^{\tau_m}) + \pi_{\delta(0),\delta(0)}\\ &\quad \times(\gamma^{\tau_{M\delta(0)}} - \gamma^{\tau_{m\delta(0)}}))/(1-\gamma)\big)/(1-\gamma)\\ &\quad + \lambda_{\max}(\tilde{Q}_2)\big(1 + \bar{\pi}_{\delta(0)}(d_M - d_m)\\ &\quad + \pi_{\delta(0),\delta(0)}(d_{M\delta(0)} - d_{m\delta(0)}) - \gamma^{d_{M\delta(0)}}\\ &\quad + (\bar{\pi}_{\delta(0)}(\gamma^{d_M} - \gamma^{d_m}) + \pi_{\delta(0),\delta(0)}\\ &\quad \times(\gamma^{d_{M\delta(0)}} - \gamma^{d_{m\delta(0)}}))/(1-\gamma)\big)/(1-\gamma)\\ &\quad + \lambda_{\max}(P_3)\lambda_{\max}(\bar{R})\big(1 + \tau_{M\delta(0)} - \tau_{m\delta(0)}\\ &\quad + (\gamma^{\tau_{M\delta(0)}+1} - \gamma^{\tau_{m\delta(0)}})/(1-\gamma)\big)/(1-\gamma)\\ &\quad + \lambda_{\max}(Q_3)\lambda_{\max}(\bar{R})\big(1 + d_{M\delta(0)} - d_{m\delta(0)}\\ &\quad + (\gamma^{d_{M\delta(0)}+1} - \gamma^{d_{m\delta(0)}})/(1-\gamma)\big)/(1-\gamma).\end{aligned}$$

Proof: In order to simplify to the expression, define $\delta(k) \triangleq q$, $\delta(k+1) \triangleq r$, $\vartheta(k) \triangleq \iota$, and $\vartheta(k+1) \triangleq \jmath$. Then, a Lyapunov–Krasovskii functional candidate is defined for the EES as follows:

$$V(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \iota) = \sum_{l=1}^{4}V_l(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \iota) \tag{19}$$

where

$$\begin{aligned}&V_1(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \iota)\\ &\quad = \boldsymbol{e}_x(k)^T P_{q,\iota}\boldsymbol{e}_x(k) + \boldsymbol{e}_y(k)^T Q_{q,\iota}\boldsymbol{e}_y(k)\\ &V_2(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \iota)\\ &\quad = \sum_{i=k-\tau_q(k)}^{k-1}\gamma^{k-i-1}\boldsymbol{e}_x(i)^T P_2\boldsymbol{e}_x(i)\\ &\qquad + \bar{\pi}_q\sum_{i=k-\tau_M+1}^{k-\tau_m}\sum_{j=i}^{k-1}\gamma^{k-j-1}\boldsymbol{e}_x(j)^T P_2\boldsymbol{e}_x(j)\\ &\qquad + \pi_{q,q}\sum_{i=k-\tau_{Mq}+1}^{k-\tau_{mq}}\sum_{j=i}^{k-1}\gamma^{k-j-1}\boldsymbol{e}_x(j)^T P_2\boldsymbol{e}_x(j)\\ &V_3(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \iota)\\ &\quad = \sum_{i=k-d_q(k)}^{k-1}\gamma^{k-i-1}\boldsymbol{e}_y(i)^T Q_2\boldsymbol{e}_y(i)\end{aligned}$$

$$+\bar{\pi}_q \sum_{i=k-d_M+1}^{k-d_m} \sum_{j=i}^{k-1} \gamma^{k-j-1} \boldsymbol{e}_y(j)^T Q_2 \boldsymbol{e}_y(j)$$
$$+\pi_{q,q} \sum_{i=k-d_{Mq}+1}^{k-d_{mq}} \sum_{j=i}^{k-1} \gamma^{k-j-1} \boldsymbol{e}_y(j)^T Q_2 \boldsymbol{e}_y(j)$$

$$V_4(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath) = \sum_{i=k-\tau_{Mq}}^{k-\tau_{mq}} \sum_{j=i}^{k-1} \gamma^{k-j-1} \boldsymbol{\hbar}_1(\boldsymbol{e}_x(j))^T P_3 \boldsymbol{\hbar}_1(\boldsymbol{e}_x(j)) + \sum_{i=k-d_{Mq}}^{k-d_{mq}} \sum_{j=i}^{k-1} \gamma^{k-j-1} \boldsymbol{\hbar}_2(\boldsymbol{e}_y(j))^T Q_3 \boldsymbol{\hbar}_2(\boldsymbol{e}_y(j)). \quad (20)$$

Then, the difference of the Lyapunov–Krasovskii functional candidate in the mean sense is defined as

$$\begin{aligned}
\mathbb{E}\{\Delta V(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\}
&\triangleq \sum_{l=1}^{4} \mathbb{E}\{\Delta V_l(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\} \\
&\triangleq \sum_{l=1}^{4} \big(\mathbb{E}\{V_l(k+1, \boldsymbol{x}(k+1), \boldsymbol{y}(k+1), r, \jmath)\} \\
&\quad - \gamma \mathbb{E}\{V_l(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\}\big). \qquad (21)
\end{aligned}$$

From (19) to (21), we have

$$\begin{aligned}
\mathbb{E}\{\Delta V_1(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\}
&= \mathbb{E}\big\{\boldsymbol{e}_x(k+1)^T \bar{P}_{r,\jmath} \boldsymbol{e}_x(k+1) \\
&\quad + \boldsymbol{e}_y(k+1)^T \bar{Q}_{r,\jmath} \boldsymbol{e}_y(k+1) \\
&\quad - \gamma \boldsymbol{e}_x(k)^T P_{q,\imath} \boldsymbol{e}_x(k) - \gamma \boldsymbol{e}_y(k)^T Q_{q,\imath} \boldsymbol{e}_y(k)\big\}. \qquad (22)
\end{aligned}$$

Also, $\mathbb{E}\{\Delta V_2(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\}$ is as follows:

$$\begin{aligned}
&\mathbb{E}\{\Delta V_2(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\} \\
&= \mathbb{E}\Bigg\{\pi_{q,q}\Bigg(\sum_{i=k-\tau_q(k+1)+1}^{k} \gamma^{k-i} \boldsymbol{e}_x(i)^T P_2 \boldsymbol{e}_x(i) \\
&\quad - \sum_{i=k-\tau_q(k)}^{k-1} \gamma^{k-i} \boldsymbol{e}_x(i)^T P_2 \boldsymbol{e}_x(i)\Bigg) \\
&\quad + \sum_{r=1, r\neq q}^{\mathfrak{S}} \pi_{q,r}\Bigg(\sum_{i=k-\tau_r(k+1)+1}^{k} \gamma^{k-i} \boldsymbol{e}_x(i)^T \\
&\quad \times P_2 \boldsymbol{e}_x(i) - \sum_{i=k-\tau_q(k)}^{k-1} \gamma^{k-i} \boldsymbol{e}_x(i)^T P_2 \boldsymbol{e}_x(i)\Bigg) \\
&\quad + \bar{\pi}_q\Bigg((\tau_M - \tau_m)\boldsymbol{e}_x(k)^T P_2 \boldsymbol{e}_x(k) \\
&\quad - \sum_{i=k-\tau_M+1}^{k-\tau_m} \gamma^{k-i} \boldsymbol{e}_x(i)^T P_2 \boldsymbol{e}_x(i)\Bigg) \\
&\quad + \pi_{q,q}\Bigg((\tau_{Mq} - \tau_{mq})\boldsymbol{e}_x(k)^T P_2 \boldsymbol{e}_x(k) \\
&\quad - \sum_{i=k-\tau_{Mq}+1}^{k-\tau_{mq}} \gamma^{k-i} \boldsymbol{e}_x(i)^T P_2 \boldsymbol{e}_x(i)\Bigg)\Bigg\} \\
&\le \mathbb{E}\big\{(\bar{\pi}_q(\tau_M - \tau_m) + 1)\boldsymbol{e}_x(k)^T P_2 \boldsymbol{e}_x(k) \\
&\quad + \pi_{q,q}(\tau_{Mq} - \tau_{mq})\boldsymbol{e}_x(k)^T P_2 \boldsymbol{e}_x(k) \\
&\quad - \gamma^{\tau_{mq}} \boldsymbol{e}_x(k-\tau_q(k))^T P_2 \boldsymbol{e}_x(k-\tau_q(k))\big\}. \qquad (23)
\end{aligned}$$

Similar to (23), it follows that

$$\begin{aligned}
&\mathbb{E}\{\Delta V_3(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\} \\
&\le \mathbb{E}\{(\bar{\pi}_q(d_M - d_m) + 1)\boldsymbol{e}_y(k)^T Q_2 \boldsymbol{e}_y(k) \\
&\quad + \pi_{q,q}(d_{Mq} - \tau_{mq})\boldsymbol{e}_y(k)^T Q_2 \boldsymbol{e}_y(k) \\
&\quad - \gamma^{d_{mq}} \boldsymbol{e}_y(k-d_q(k))^T Q_2 \boldsymbol{e}_y(k-d_q(k))\}. \qquad (24)
\end{aligned}$$

Finally, the condition (25) holds

$$\begin{aligned}
&\mathbb{E}\{\Delta V_4(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\} \\
&= \mathbb{E}\Bigg\{\sum_{i=k-\tau_{Mq}}^{k-\tau_{mq}} \sum_{j=i+1}^{k} \gamma^{k-j} \boldsymbol{\hbar}_1(\boldsymbol{e}_x(j))^T P_3 \boldsymbol{\hbar}_1(\boldsymbol{e}_x(j)) \\
&\quad - \sum_{i=k-\tau_{Mq}}^{k-\tau_{mq}} \sum_{j=i}^{k-1} \gamma^{k-j} \boldsymbol{\hbar}_1(\boldsymbol{e}_x(j))^T P_3 \boldsymbol{\hbar}_1(\boldsymbol{e}_x(j)) \\
&\quad + \sum_{i=k-d_{Mq}}^{k-d_{mq}} \sum_{j=i+1}^{k} \gamma^{k-j} \boldsymbol{\hbar}_2(\boldsymbol{e}_y(j))^T Q_3 \boldsymbol{\hbar}_2(\boldsymbol{e}_y(j)) \\
&\quad - \sum_{i=k-d_{Mq}}^{k-d_{mq}} \sum_{j=i}^{k-1} \gamma^{k-j} \boldsymbol{\hbar}_2(\boldsymbol{e}_y(j))^T Q_3 \boldsymbol{\hbar}_2(\boldsymbol{e}_y(j))\Bigg\} \\
&\le \mathbb{E}\{(\tau_{Mq} - \tau_{mq} + 1)\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k))^T P_3 \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k)) \\
&\quad - \gamma^{\tau_{mq}} \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k)))^T P_3 \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k))) \\
&\quad + (d_{Mq} - d_{mq} + 1)\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k))^T Q_3 \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k)) \\
&\quad - \gamma^{d_{mq}} \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k)))^T Q_3 \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k)))\}. \qquad (25)
\end{aligned}$$

Considering the EES (12) and the formulas (22)–(25), it yields

$$\begin{aligned}
&\mathbb{E}\{\Delta V(k, \boldsymbol{x}(k), \boldsymbol{y}(k), q, \imath)\} \\
&\le \mathbb{E}\big\{\boldsymbol{e}_x(k)^T \bar{A}_{q,\imath}^T \bar{P}_{r,\jmath} \bar{A}_{q,\imath} \boldsymbol{e}_x(k) \\
&\quad + \hbar_2(\boldsymbol{e}_y(k-d_q(k)))^T B_q^T \bar{P}_{r,\jmath} B_q \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k))) \\
&\quad + 2\boldsymbol{e}_x(k)^T \bar{A}_{q,\imath}^T \bar{P}_{r,\jmath} B_q \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k))) \\
&\quad + 2\boldsymbol{e}_x(k)^T \bar{A}_{q,\imath}^T \bar{P}_{r,\jmath} \bar{E}_{q,\imath} \boldsymbol{v}_1(k) \\
&\quad + 2\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k)))^T B_q^T \bar{P}_{r,\jmath} \bar{E}_{q,\imath} \boldsymbol{v}_1(k) \\
&\quad + \boldsymbol{v}_1(k)^T \bar{E}_{q,\imath}^T \bar{P}_{r,\jmath} \bar{E}_{q,\imath} \boldsymbol{v}_1(k) - \gamma \boldsymbol{e}_x(k)^T P_{q,\imath} \boldsymbol{e}_x(k) \\
&\quad + \boldsymbol{e}_y(k)^T \bar{C}_{q,\imath}^T \bar{Q}_{r,\jmath} \bar{C}_{q,\imath} \boldsymbol{e}_y(k) \\
&\quad + \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k)))^T D_q^T \bar{Q}_{r,\jmath} D_q \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k))) \\
&\quad + 2\boldsymbol{e}_y(k)^T \bar{C}_{q,\imath}^T \bar{Q}_{r,\jmath} D_q \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k))) \\
&\quad + 2\boldsymbol{e}_y(k)^T \bar{C}_{q,\imath}^T \bar{Q}_{r,\jmath} \bar{F}_{q,\imath} \boldsymbol{v}_2(k) \\
&\quad + 2\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k)))^T D_q^T \bar{Q}_{r,\jmath} \bar{F}_{q,\imath} \boldsymbol{v}_2(k) \\
&\quad + \boldsymbol{v}_2(k)^T \bar{F}_{q,\imath}^T \bar{Q}_{r,\jmath} \bar{F}_{q,\imath} \boldsymbol{v}_2(k) - \gamma \boldsymbol{e}_y(k)^T Q_{q,\imath} \boldsymbol{e}_y(k) \\
&\quad + \bar{\pi}_{\tau,q} \boldsymbol{e}_x(k)^T P_2 \boldsymbol{e}_x(k) + \bar{\pi}_{d,q} \boldsymbol{e}_y(k)^T Q_2 \boldsymbol{e}_y(k) \\
&\quad + \pi_{q,q}(\tau_{Mq} - \tau_{mq})\boldsymbol{e}_x(k)^T P_2 \boldsymbol{e}_x(k) \\
&\quad - \gamma^{\tau_{mq}} \boldsymbol{e}_x(k-\tau_q(k))^T P_2 \boldsymbol{e}_x(k-\tau_q(k)) \\
&\quad + \pi_{q,q}(d_{Mq} - d_{mq})\boldsymbol{e}_y(k)^T Q_2 \boldsymbol{e}_y(k)
\end{aligned}$$

$$
\begin{aligned}
&-\gamma^{d_{mq}}\boldsymbol{e}_y(k-d_q(k))^T Q_2\boldsymbol{e}_y(k-d_q(k))\\
&+(\tau_{Mq}-\tau_{mq}+1)\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k))^T P_3\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k))\\
&-\gamma^{\tau_{mq}}\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k)))^T P_3\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k)))\\
&+(d_{Mq}-d_{mq}+1)\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k))^T Q_3\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k))\\
&-\gamma^{d_{mq}}\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k)))^T Q_3\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k)))\}.
\end{aligned}\tag{26}
$$

Based on Assumption 1, there exist diagonal matrices $\boldsymbol{\epsilon}_1 \succ 0$, $\boldsymbol{\epsilon}_2 \succ 0$, $\boldsymbol{\varepsilon}_1 \succ 0$, and $\boldsymbol{\varepsilon}_2 \succ 0$ such that the inequalities (27) are guaranteed

$$
\begin{aligned}
&\boldsymbol{\eta}_{17}(k)^T\begin{bmatrix}0 & \boldsymbol{\epsilon}_1\boldsymbol{\ell}_2\\ * & -\boldsymbol{\epsilon}_1\end{bmatrix}\boldsymbol{\eta}_{17}(k)\geq 0\\
&\boldsymbol{\eta}_{53}(k)^T\begin{bmatrix}0 & \boldsymbol{\varepsilon}_1\boldsymbol{\ell}_2\\ * & -\boldsymbol{\varepsilon}_1\end{bmatrix}\boldsymbol{\eta}_{53}(k)\geq 0\\
&\boldsymbol{\eta}_{28}(k)^T\begin{bmatrix}0 & \boldsymbol{\epsilon}_2\boldsymbol{\ell}_2\\ * & -\boldsymbol{\epsilon}_2\end{bmatrix}\boldsymbol{\eta}_{28}(k)\geq 0\\
&\boldsymbol{\eta}_{64}(k)^T\begin{bmatrix}0 & \boldsymbol{\varepsilon}_2\boldsymbol{\ell}_2\\ * & -\boldsymbol{\varepsilon}_2\end{bmatrix}\boldsymbol{\eta}_{64}(k)\geq 0
\end{aligned}\tag{27}
$$

where

$$
\begin{aligned}
\boldsymbol{\eta}_{17}(k)&=\begin{bmatrix}\boldsymbol{e}_x(k)\\ \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k))\end{bmatrix},\quad \boldsymbol{\eta}_{28}(k)=\begin{bmatrix}\boldsymbol{e}_x(k-\tau_\delta(k))\\ \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_\delta(k)))\end{bmatrix}\\
\boldsymbol{\eta}_{53}(k)&=\begin{bmatrix}\boldsymbol{e}_y(k)\\ \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k))\end{bmatrix},\quad \boldsymbol{\eta}_{64}(k)=\begin{bmatrix}\boldsymbol{e}_y(k-d_\delta(k))\\ \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_\delta(k)))\end{bmatrix}.
\end{aligned}
$$

In view of (26), substituting the left-hand side of (27) into the right-hand side of (26) gives

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(k,\boldsymbol{x}(k),\boldsymbol{y}(k),q,\iota)\}\\
&\quad\leq \mathbb{E}\{\boldsymbol{\eta}(k)^T\Omega\boldsymbol{\eta}(k)\\
&\qquad+\boldsymbol{v}_1(k)^T\mathcal{S}_1\boldsymbol{v}_1(k)+\boldsymbol{v}_2(k)^T\mathcal{S}_2\boldsymbol{v}_2(k)\}\\
&\quad=\mathbb{E}\{\boldsymbol{\eta}(k)^T\Omega\boldsymbol{\eta}(k)+\boldsymbol{v}(k)^T\mathcal{S}\boldsymbol{v}(k)\}
\end{aligned}
$$

where

$$
\Omega=\begin{bmatrix}
\Omega_{11} & 0 & 0 & \Omega_{14} & 0 & 0 & \boldsymbol{\epsilon}_1\boldsymbol{\ell}_2 & 0 & \Omega_{19}\\
* & \Omega_{22} & 0 & 0 & 0 & 0 & 0 & \boldsymbol{\epsilon}_2\boldsymbol{\ell}_2 & 0\\
* & * & \Omega_{33} & 0 & \boldsymbol{\ell}_2^T\boldsymbol{\varepsilon}_1^T & 0 & 0 & 0 & 0\\
* & * & * & \Omega_{44} & 0 & \boldsymbol{\ell}_2^T\boldsymbol{\varepsilon}_2^T & 0 & 0 & \Omega_{49}\\
* & * & * & * & \Omega_{55} & 0 & 0 & \Omega_{58} & \Omega_{59}\\
* & * & * & * & * & \Omega_{66} & 0 & 0 & 0\\
* & * & * & * & * & * & \Omega_{77} & 0 & 0\\
* & * & * & * & * & * & * & \Omega_{88} & \Omega_{89}\\
* & * & * & * & * & * & * & * & \Omega_{99}
\end{bmatrix}\tag{28}
$$

with

$$
\begin{aligned}
\boldsymbol{\eta}(k)&=\big[\boldsymbol{e}_x(k)^T\ \boldsymbol{e}_x(k-\tau_q(k))^T\ \boldsymbol{\hbar}_2(\boldsymbol{e}_y(k))^T\\
&\qquad\boldsymbol{\hbar}_2(\boldsymbol{e}_y(k-d_q(k)))^T\ \boldsymbol{e}_y(k)^T\ \boldsymbol{e}_y(k-d_q(k))^T\\
&\qquad\boldsymbol{\hbar}_1(\boldsymbol{e}_x(k))^T\ \boldsymbol{\hbar}_1(\boldsymbol{e}_x(k-\tau_q(k)))^T\ \boldsymbol{v}(k)^T\big]^T\\
\boldsymbol{v}(k)&=\begin{bmatrix}\boldsymbol{v}_1(k)^T & \boldsymbol{v}_2(k)^T\end{bmatrix}^T,\quad \mathcal{S}=\mathrm{diag}\{\mathcal{S}_1,\mathcal{S}_2\}\\
\Omega_{11}&=\bar{A}_{q,\iota}^T\bar{P}_{r,\jmath}\bar{A}_{q,\iota}-\gamma P_{q,\iota}+\bar{\pi}_{\tau,q}P_2+\pi_{q,q}(\tau_{Mq}-\tau_{mq})P_2\\
\Omega_{14}&=\bar{A}_{q,\iota}^T\bar{P}_{r,\jmath}B_q,\quad \Omega_{49}=\begin{bmatrix}B_q^T\bar{P}_{r,\jmath}\bar{E}_{q,\iota} & 0\end{bmatrix}\\
\Omega_{19}&=\begin{bmatrix}\bar{A}_{q,\iota}^T\bar{P}_{r,\jmath}\bar{E}_{q,\iota} & 0\end{bmatrix},\quad \Omega_{59}=\begin{bmatrix}0 & \bar{C}_{q,\iota}^T\bar{Q}_{r,\jmath}\bar{F}_{q,\iota}\end{bmatrix}\\
\Omega_{22}&=-\gamma^{\tau_{mq}}P_2,\ \ \Omega_{66}=-\gamma^{d_{mq}}Q_2\\
\Omega_{33}&=(d_{Mq}-d_{mq}+1)Q_3-\boldsymbol{\varepsilon}_1\\
\Omega_{77}&=(\tau_{Mq}-\tau_{mq}+1)P_3-\boldsymbol{\epsilon}_1\\
\Omega_{44}&=B_q^T\bar{P}_{r,\jmath}B_q-\gamma^{d_{mq}}Q_3-\boldsymbol{\varepsilon}_2\\
\Omega_{55}&=\bar{C}_{q,\iota}^T\bar{Q}_{r,\jmath}\bar{C}_{q,\iota}-\gamma Q_{q,\iota}+\bar{\pi}_{d,q}Q_2\\
&\quad+\pi_{q,q}(d_{Mq}-d_{mq})Q_2\\
\Omega_{58}&=\bar{C}_{q,\iota}^T\bar{Q}_{r,\jmath}D_q,\ \ \Omega_{89}=\begin{bmatrix}0 & D_q^T\bar{Q}_{r,\jmath}\bar{F}_{q,\iota}\end{bmatrix}\\
\Omega_{88}&=D_q^T\bar{Q}_{r,\jmath}D_q-\gamma^{\tau_{mq}}P_3-\boldsymbol{\epsilon}_2\\
\Omega_{99}&=\begin{bmatrix}\bar{E}_{q,\iota}^T\bar{P}_{r,\jmath}\bar{E}_{q,\iota} & 0\\ * & \bar{F}_{q,\iota}^T\bar{Q}_{r,\jmath}\bar{F}_{q,\iota}\end{bmatrix}-\mathcal{S}.
\end{aligned}
$$

Employing the Schur complement lemma to (15), it follows $\Omega \prec 0$, which implies

$$
\mathbb{E}\{\Delta\ V(k,\boldsymbol{x}(k),\boldsymbol{y}(k),q,\iota)\}<\boldsymbol{v}(k)\mathcal{S}\boldsymbol{v}(k).\tag{29}
$$

By using the iterative method to (29), we obtain the following condition:

$$
\begin{aligned}
&\mathbb{E}\{V(k,\boldsymbol{x}(k),\boldsymbol{y}(k),\delta(k),\vartheta(k))\}\\
&\quad<\gamma^k\mathbb{E}\{V(0,\boldsymbol{x}(0),\boldsymbol{y}(0),\delta(0),\vartheta(0))\}\\
&\qquad+\lambda_{\max}(\mathcal{S})\sum_{i=0}^{k-1}\gamma^{k-i-1}\boldsymbol{v}(i)^T\boldsymbol{v}(i)\\
&\quad<\gamma^k\big(\mathbb{E}\{V(0,\boldsymbol{x}(0),\boldsymbol{y}(0),\delta(0),\vartheta(0))\}+\lambda_{\max}(\mathcal{S})c^2\big).
\end{aligned}\tag{30}
$$

According to (5) of Assumption 1, (13), and (19), we achieve

$$
\begin{aligned}
&\mathbb{E}\{V(0,\boldsymbol{x}(0),\boldsymbol{y}(0),\delta(0),\vartheta(0))\}\\
&\leq\Bigg(\lambda_{\max}(\tilde{P}_{\delta(0),\vartheta(0)})+\lambda_{\max}(\tilde{Q}_{\delta(0),\vartheta(0)})\\
&\quad+\lambda_{\max}(\tilde{P}_2)\Bigg(\bar{\pi}_{\delta(0)}\sum_{i=-\tau_M+1}^{-\tau_m}\sum_{j=i}^{-1}\gamma^{-j-1}\\
&\qquad+\pi_{\delta(0),\delta(0)}\sum_{i=-\tau_{M\delta(0)}+1}^{-\tau_{m\delta(0)}}\sum_{j=i}^{-1}\gamma^{-j-1}\\
&\qquad+\sum_{i=-\tau_{M\delta(0)}}^{-1}\gamma^{-i-1}\Bigg)+\lambda_{\max}(\tilde{Q}_2)\\
&\quad\times\Bigg(\bar{\pi}_{\delta(0)}\sum_{i=-d_M+1}^{-d_m}\sum_{j=i}^{-1}\gamma^{-j-1}+\sum_{i=-d_{M\delta(0)}}^{-1}\gamma^{-i-1}\\
&\qquad+\pi_{\delta(0),\delta(0)}\sum_{i=-d_{M\delta(0)}+1}^{-d_{m\delta(0)}}\sum_{j=i}^{-1}\gamma^{-j-1}\Bigg)\\
&\quad+\ \lambda_{\max}(P_3)\lambda_{\max}(\bar{R})\sum_{i=-\tau_{M\delta(0)}}^{-\tau_{m\delta(0)}}\sum_{j=i}^{-1}\gamma^{-j-1}\\
&\quad+\lambda_{\max}(Q_3)\lambda_{\max}(\bar{R})\sum_{i=-d_{M\delta(0)}}^{-d_{m\delta(0)}}\sum_{j=i}^{-1}\gamma^{-j-1}\Bigg)\delta_1^2\\
&=\lambda_{\delta(0),\vartheta(0)}\delta_1^2.
\end{aligned}\tag{31}
$$

Combining (30) with (31), the inequality (32) holds

$$
\begin{aligned}
&\mathbb{E}\{V(k,\boldsymbol{x}(k),\boldsymbol{y}(k),\delta(k),\vartheta(k))\}\\
&\qquad<\gamma^k\Big(\max_{\delta(0)\in\mathfrak{S},\vartheta(0)\in S}\{\lambda_{\delta(0),\vartheta(0)}\}\delta_1^2+\lambda_{\max}(\mathcal{S})c^2\Big).
\end{aligned}\tag{32}
$$

In addition, from (16), (17), and (19), we obtain

$$l_0\mathbb{E}\{\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k)+\boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k)\} \leq \mathbb{E}\{V(k,\boldsymbol{x}(k),\boldsymbol{y}(k),\delta(k),\vartheta(k))\}. \tag{33}$$

Then, it follows from (32) and (33) that

$$\mathbb{E}\{\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k)+\boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k)\} < \frac{\gamma^k(\max_{\delta(0)\in\mathfrak{S},\vartheta(0)\in S}\{\lambda_{\delta(0),\vartheta(0)}\}\delta_1^2+\lambda_{\max}(\mathcal{S})c^2)}{l_0}.$$

From the inequality (18), the condition $\mathbb{E}\{\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k)+\boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k)\}<\delta_2^2$ holds, which means that the inequality (14) in Definition 1 is satisfied, that is, the EES (12) meets the SFTB. ■

Remark 2: From Theorem 1, a new Lyapunov–Krasovskii function is established. By using the Lyapunov theory, sufficient conditions are derived to guarantee the SFTB for the EES (12), and the inequality constraint conditions of bound δ_2^2 are obtained simultaneously.

Since Theorem 1 contains nonlinear terms, the controller gains cannot be obtained by the linear matrix inequality (LMI) technology directly. Thus, a new theorem as follows is proposed to design the controller gains.

Theorem 2: Given a scalar $\gamma>1$, the EES (12) is SFTB, if there exist matrices $P_{q,\iota}\succ 0$, $Q_{q,\iota}\succ 0$, $P_2\succ 0$, $P_3\succ 0$, $Q_2\succ 0$, $Q_3\succ 0$, $\boldsymbol{\epsilon}_1\succ 0$, $\boldsymbol{\epsilon}_2\succ 0$, $\boldsymbol{\varepsilon}_1\succ 0$, $\boldsymbol{\varepsilon}_2\succ 0$, $\mathcal{S}_1\triangleq\begin{bmatrix}\mathcal{S}_{1,1} & 0\\ * & \mathcal{S}_{1,2}\end{bmatrix}\succ 0$, $\mathcal{S}_2\triangleq\begin{bmatrix}\mathcal{S}_{2,1} & 0\\ * & \mathcal{S}_{2,2}\end{bmatrix}\succ 0$, G_1, G_2, $\bar{K}_x(1)$, and $\bar{K}_y(1)$, and positive scalars l_0, λ_P, λ_{P_2}, λ_{P_3}, $\lambda_{\bar{P}_3}$, λ_Q, λ_{Q_2}, λ_{Q_3}, $\lambda_{\bar{Q}_3}$, and λ_s, such that the following LMIs hold for $\forall q\in\mathfrak{S}$, $\iota\in S$, and $\theta\in\Theta\triangleq\{1,2,3,4\}$:

$$\Xi(\theta)=\begin{bmatrix}\Psi_{11} & \Psi_{12} & 0 & 0 & \Xi_{15}(\theta) & 0\\ * & \Psi_{22} & 0 & 0 & 0 & \Xi_{26}(\theta)\\ * & * & -\mathcal{S}_1 & 0 & \Xi_{35}(\theta) & 0\\ * & * & * & -\mathcal{S}_2 & 0 & \Xi_{46}(\theta)\\ * & * & * & * & \Xi_{55} & 0\\ * & * & * & * & * & \Xi_{66}\end{bmatrix}\prec 0 \tag{34}$$

$$\begin{aligned}
l_0R\prec P_{q,\iota}\prec\lambda_P R,&\ \ P_2\prec\lambda_{P_2}R\\
P_3\prec\lambda_{P_3}I,&\ \ l_0R\prec Q_{q,\iota}\prec\lambda_Q R\\
Q_2\prec\lambda_{Q_2}R,&\ \ Q_3\prec\lambda_{Q_3}I,\ \ \mathcal{S}\prec\lambda_s I\\
\lambda_{P_3}\boldsymbol{\ell}_1^T\boldsymbol{\ell}_1\prec\lambda_{\bar{P}_3}R,&\ \ \lambda_{Q_3}\boldsymbol{\ell}_1^T\boldsymbol{\ell}_1\prec\lambda_{\bar{Q}_3}R
\end{aligned} \tag{35}$$

$$\gamma^N\left(\max_{\delta(0)\in\mathfrak{S},\vartheta(0)\in S}\{\bar{\lambda}_{\delta(0),\vartheta(0)}\}\delta_1^2+\lambda_s c^2\right)\leq l_0\delta_2^2 \tag{36}$$

with

$$\begin{aligned}
\Xi_{15}(\theta)&=\left[G_1A_q-\bar{K}_x(\theta)L_{x,\iota}\quad 0\quad 0\quad G_1B_q\right]^T\\
\Xi_{26}(\theta)&=\left[G_2C_q-\bar{K}_y(\theta)L_{y,\iota}\quad 0\quad 0\quad G_2D_q\right]^T\\
\Xi_{35}(\theta)&=\left[G_1E_q\quad -\bar{K}_x(\theta)M_{x,\iota}\right]^T\\
\Xi_{46}(\theta)&=\left[G_2F_q\quad -\bar{K}_y(\theta)M_{y,\iota}\right]^T\\
\bar{K}_x(2)&=\bar{K}_x(1),\ \ \bar{K}_y(3)=\bar{K}_y(1)\\
\bar{K}_x(3)&=\bar{K}_x(4)=\bar{K}_x(1)+G_1\urcorner\\
\bar{K}_y(2)&=\bar{K}_y(4)=\bar{K}_y(1)+G_2\Gamma\\
\Xi_{55}&=\bar{P}_{r,j}-G_1-G_1^T,\quad \Xi_{66}=\bar{Q}_{r,j}-G_2-G_2^T
\end{aligned}$$

$$\begin{aligned}
\bar{\lambda}_{\delta(0),\vartheta(0)}=\ &\lambda_P+\lambda_{P_2}\big(1+\bar{\pi}_{\delta(0)}(\tau_M-\tau_m)\\
&+\pi_{\delta(0),\delta(0)}(\tau_{M\delta(0)}-\tau_{m\delta(0)})-\gamma^{\tau_{M\delta(0)}}\\
&+(\bar{\pi}_{\delta(0)}(\gamma^{\tau_M}-\gamma^{\tau_m})+\pi_{\delta(0),\delta(0)}\\
&\times(\gamma^{\tau_{M\delta(0)}}-\gamma^{\tau_{m\delta(0)}}))/(1-\gamma)\big)/(1-\gamma)\\
&+\lambda_Q+\lambda_{Q_2}\big(1+\bar{\pi}_{\delta(0)}(d_M-d_m)\\
&+\pi_{\delta(0),\delta(0)}(d_{M\delta(0)}-d_{m\delta(0)})-\gamma^{d_{M\delta(0)}}\\
&+(\bar{\pi}_{\delta(0)}(\gamma^{d_M}-\gamma^{d_m})+\pi_{\delta(0),\delta(0)}\\
&\times(\gamma^{d_{M\delta(0)}}-\gamma^{d_{m\delta(0)}}))/(1-\gamma)\big)/(1-\gamma)\\
&+\lambda_{\bar{P}_3}\big(1+\tau_{M\delta(0)}-\tau_{m\delta(0)}+(\gamma^{\tau_{M\delta(0)}+1}\\
&-\gamma^{\tau_{m\delta(0)}})/(1-\gamma)\big)/(1-\gamma)\\
&+\lambda_{\bar{Q}_3}\big(1+d_{M\delta(0)}-d_{m\delta(0)}+(\gamma^{d_{M\delta(0)}+1}\\
&-\gamma^{d_{m\delta(0)}})/(1-\gamma)\big)/(1-\gamma).
\end{aligned}$$

Then, the vertices of estimator gains are given as

$$\begin{aligned}
K_{x,1}&=G_1^{-1}\bar{K}_x(1),\quad K_{y,1}=G_2^{-1}\bar{K}_y(1)\\
K_{x,2}&=K_{x,1}+\urcorner,\quad K_{y,2}=K_{y,1}+\Gamma.
\end{aligned}$$

Proof: Considering the inequalities

$$\begin{aligned}
(\bar{P}_{r,j}-G_1)\bar{P}_{r,j}^{-1}(\bar{P}_{r,j}-G_1)^T&\succeq 0\\
(\bar{Q}_{r,j}-G_2)\bar{Q}_{r,j}^{-1}(\bar{Q}_{r,j}-G_2)^T&\succeq 0
\end{aligned}$$

we obtain

$$\begin{aligned}
\bar{P}_{r,j}-G_1-G_1^T&\succeq -G_1\bar{P}_{r,j}^{-1}G_1^T\\
\bar{Q}_{r,j}-G_2-G_2^T&\succeq -G_2\bar{Q}_{r,j}^{-1}G_2^T.
\end{aligned} \tag{37}$$

Then, define the matrices $\bar{K}_x(1)\triangleq G_1K_{x,1}$ and $\bar{K}_y(1)\triangleq G_2K_{y,1}$. Substituting the definition of matrices $\bar{K}_x(1)$ and $\bar{K}_y(1)$ and the inequalities (37) into the inequality (34) and premultiplying and postmultiplying the obtained matrix inequalities with $\mathrm{diag}\{I,\ I,\ I,\ I,\ G_1^{-1},\ G_2^{-1}\}$ and its transposition, the following condition is obtained:

$$\Phi(\theta)=\begin{bmatrix}\Psi_{11} & \Psi_{12} & 0 & 0 & \Phi_{15}(\theta) & 0\\ * & \Psi_{22} & 0 & 0 & 0 & \Phi_{26}(\theta)\\ * & * & -\mathcal{S}_1 & 0 & \Phi_{35}(\theta) & 0\\ * & * & * & -\mathcal{S}_2 & 0 & \Phi_{46}(\theta)\\ * & * & * & * & -\bar{P}_{r,j}^{-1} & 0\\ * & * & * & * & * & -\bar{Q}_{r,j}^{-1}\end{bmatrix}\prec 0$$

where

$$\begin{aligned}
\Phi_{15}(\theta)&=\left[A_q-K_x(\theta)L_{x,\iota}\ \ 0\ \ 0\ \ B_q\right]^T\\
\Phi_{26}(\theta)&=\left[C_q-K_y(\theta)L_{y,\iota}\ \ 0\ \ 0\ \ D_q\right]^T\\
\Phi_{35}(\theta)&=\left[E_q\ \ -K_x(\theta)M_{x,\iota}\right]^T\\
\Phi_{46}(\theta)&=\left[F_q\ \ -K_y(\theta)M_{y,\iota}\right]^T
\end{aligned}$$

with

$$\begin{aligned}
K_x(1)&=K_x(2)=K_{x,1},\ \ K_x(3)=K_x(4)=K_{x,2}\\
K_y(1)&=K_y(3)=K_{y,1},\ \ K_y(2)=K_y(4)=K_{y,2}.
\end{aligned}$$

In view of the definition (10), we have

$$\alpha\beta\Phi(1)+\alpha(1-\beta)\Phi(2)+(1-\alpha)\beta\Phi(3)+(1-\alpha)(1-\beta)\Phi(4)\prec 0$$

which implies that the condition (15) holds. In addition, from the inequalities (35) and (36), the condition (18) holds, that is, the EES (12) satisfies the SFTB based on Theorem 1. ■

IV. Illustrative Example

In this section, an illustrative example is given to demonstrate the effectiveness of the proposed estimator for discrete-time Markovian BAM NNs. Assume that $\delta(k) \in \mathfrak{S} = \{1, 2, 3\}$, $\vartheta(k) \in S = \{1, 2\}$, and $k \in [1,\ 20]$, and the parameters of Markovian BAM NNs are

$$A_1 = \mathrm{diag}\{0.70, 0.60, 0.50\}$$
$$A_2 = \mathrm{diag}\{0.70, 0.80, 0.90\}$$
$$A_3 = \mathrm{diag}\{0.50, 0.60, 0.80\}$$
$$C_1 = \mathrm{diag}\{0.70, 0.60, 0.80\}$$
$$C_2 = \mathrm{diag}\{0.80, 0.70, 0.60\}$$
$$C_3 = \mathrm{diag}\{0.70, 0.40, 0.40\}$$
$$B_1 = \begin{bmatrix} 0.25 & 0.01 & 0 \\ 0.01 & 0.28 & 0 \\ 0 & 0.01 & 0.19 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0.25 & 0.01 & 0 \\ 0.01 & 0.31 & 0 \\ 0 & 0.01 & 0.26 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} 0.15 & 0.01 & 0 \\ 0.01 & 0.25 & 0 \\ 0 & 0.01 & 0.32 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0.25 & 0.11 & 0.01 \\ 0.01 & 0.21 & 0.01 \\ 0.01 & 0.01 & 0.20 \end{bmatrix}$$
$$D_2 = \begin{bmatrix} 0.22 & 0.11 & 0.01 \\ 0.01 & 0.24 & 0.01 \\ 0.01 & 0.01 & 0.32 \end{bmatrix}$$
$$D_3 = \begin{bmatrix} 0.21 & 0.01 & 0.01 \\ 0.01 & 0.22 & 0.01 \\ 0.01 & 0.01 & 0.30 \end{bmatrix}$$
$$E_1 = \begin{bmatrix} 0.31 & 0.13 & 0.20 \end{bmatrix}^T$$
$$E_2 = \begin{bmatrix} 0.21 & 0.11 & 0.35 \end{bmatrix}^T$$
$$E_3 = \begin{bmatrix} 0.31 & 0.21 & 0.14 \end{bmatrix}^T$$
$$F_1 = \begin{bmatrix} 0.12 & 0.31 & 0.12 \end{bmatrix}^T$$
$$F_2 = \begin{bmatrix} 0.13 & 0.12 & 0.34 \end{bmatrix}^T$$
$$F_3 = \begin{bmatrix} 0.31 & 0.22 & 0.12 \end{bmatrix}^T$$
$$M_{x,1} = \begin{bmatrix} 0.08 \\ 0.10 \end{bmatrix}, \quad M_{x,2} = \begin{bmatrix} 0.09 \\ 0.10 \end{bmatrix}$$
$$M_{y,1} = \begin{bmatrix} 0.07 \\ 0.10 \end{bmatrix}, \quad M_{y,2} = \begin{bmatrix} 0.06 \\ 0.10 \end{bmatrix}$$
$$L_{x,1} = \begin{bmatrix} 0.40 & 0.10 & 0.10 \\ 0.10 & 0.20 & 0.30 \end{bmatrix}$$
$$L_{x,2} = \begin{bmatrix} 0.30 & 0.20 & 0.10 \\ 0.10 & 0.20 & 0.30 \end{bmatrix}$$
$$L_{y,1} = \begin{bmatrix} 0.40 & 0.10 & 0.20 \\ 0.10 & 0.30 & 0.50 \end{bmatrix}$$
$$L_{y,2} = \begin{bmatrix} 0.30 & 0.20 & 0.20 \\ 0.10 & 0.10 & 0.50 \end{bmatrix}.$$

The trasition probability matrix Π_δ of the Markov chain $\delta(k)$ is assumed as

$$\Pi_\delta = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}.$$

The transition probability matrices $\Pi_\vartheta^{\delta(k)}$ of the Markov chain $\vartheta(k)$ are chosen as

$$\Pi_\vartheta^1 = \begin{bmatrix} 0.80 & 0.20 \\ 0.70 & 0.30 \end{bmatrix}$$
$$\Pi_\vartheta^2 = \begin{bmatrix} 0.40 & 0.60 \\ 0.60 & 0.40 \end{bmatrix}$$
$$\Pi_\vartheta^3 = \begin{bmatrix} 0.20 & 0.80 \\ 0.30 & 0.70 \end{bmatrix}.$$

The nonlinear function are assumed as $\boldsymbol{g}(.) = [0.72\tanh(.)\ \ 0.56\tanh(.)\ \ 0.64\tanh(.)]^T$. Based on the assumption that $\delta(k) \in \mathfrak{S} = \{1, 2, 3\}$ and $\vartheta(k) \in S = \{1, 2\}$, we consider the cases that $d_1 \in [1,\ 2]$, $d_2 \in [2,\ 3]$, $d_3 \in [0,\ 1]$, $\tau_1 \in [2,\ 3]$, $\tau_2 \in [1,\ 2]$, and $\tau_3 \in [0,\ 1]$. Furthermore, suppose that $R = \mathrm{diag}\{1, 1, 1\}$, $c^2 = 0.7687$, $\delta_1^2 = 0.06$, and $\gamma = 1.02$, and the known matrices

$$\daleth = \begin{bmatrix} 0.21 & 0.23 \\ 0.24 & 0.25 \\ 0.23 & 0.28 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0.18 & 0.21 \\ 0.27 & 0.29 \\ 0.22 & 0.26 \end{bmatrix}.$$

In order to facilitate the comparison and the analysis, according to the SFTB conditions that proposed in Theorem 2, assuming that the scalar $l_0 = 1.5$, minimizing the value of δ_2^2 defined in Definition 1 by solving the LMIs, the estimator gains are given as

$$K_{x,1} = \begin{bmatrix} 0.4823 & 0.3616 \\ -0.0179 & 0.7201 \\ -0.4113 & 1.7834 \end{bmatrix}$$
$$K_{x,2} = \begin{bmatrix} 0.6923 & 0.5916 \\ 0.2221 & 0.9701 \\ -0.1813 & 2.0634 \end{bmatrix}$$
$$K_{y,1} = \begin{bmatrix} 0.9602 & -0.0464 \\ 0.2487 & 0.1266 \\ 0.3080 & 0.4411 \end{bmatrix}$$
$$K_{y,2} = \begin{bmatrix} 1.1402 & 0.1636 \\ 0.5187 & 0.4166 \\ 0.5280 & 0.7011 \end{bmatrix}.$$

The stochastic noise sequences are defined as $\boldsymbol{\omega}_1(k) = 0.10$rand, $\boldsymbol{\omega}_2(k) = 0.09$rand, $\boldsymbol{v}_1(k) = 0.08$rand, and $\boldsymbol{v}_2(k) = 0.11$rand, $k \in [1,\ 20]$, where "rand" denotes a random number between 0 and 1. Choosing the initial conditions of the BAM NNs $\boldsymbol{x}(-3) = [-0.1\ -0.1\ 0.1]^T$, $\boldsymbol{x}(-2) = [-0.1\ 0.1\ -0.1]^T$, $\boldsymbol{x}(-1) = [0.1\ \ -0.1\ 0.1]^T$, $\boldsymbol{x}(0) = [0.1\ 0.1\ 0.1]^T$, $\boldsymbol{y}(-3) = [0.1\ \ -0.1\ 0.1]^T$, $\boldsymbol{y}(-2) = [0.1\ 0.1\ \ -0.1]^T$, $\boldsymbol{y}(-1) = [0.1\ \ -0.1\ 0.0]^T$, $\boldsymbol{y}(0) = [0.1\ \ -0.1\ 0.1]^T$, and the initial conditions of the estimators $\hat{\boldsymbol{x}}(-3) = \hat{\boldsymbol{x}}(-2) = \hat{\boldsymbol{x}}(-1) = \hat{\boldsymbol{x}}(0) = 0$ and $\hat{\boldsymbol{y}}(-3) = \hat{\boldsymbol{y}}(-2) = \hat{\boldsymbol{y}}(-1) = \hat{\boldsymbol{y}}(0) = 0$, we have $\mathbb{E}\{\boldsymbol{e}_x(\imath)^T R \boldsymbol{e}_x(\imath) + \boldsymbol{e}_y(\imath)^T R \boldsymbol{e}_y(\imath)\} \le \delta_1^2 = 0.06$ for $\imath \in \{-\chi_M, -\chi_M + 1, \ldots, 0\}$. The trajectories of $\boldsymbol{x}(k)$ and

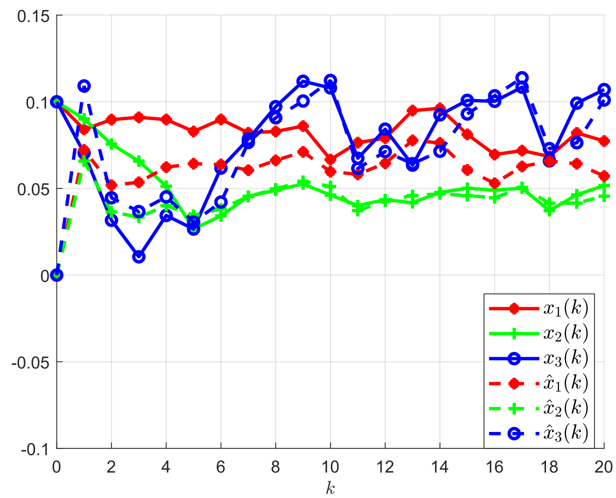

Fig. 2. Trajectories of $\boldsymbol{x}(k)$ and its estimation.

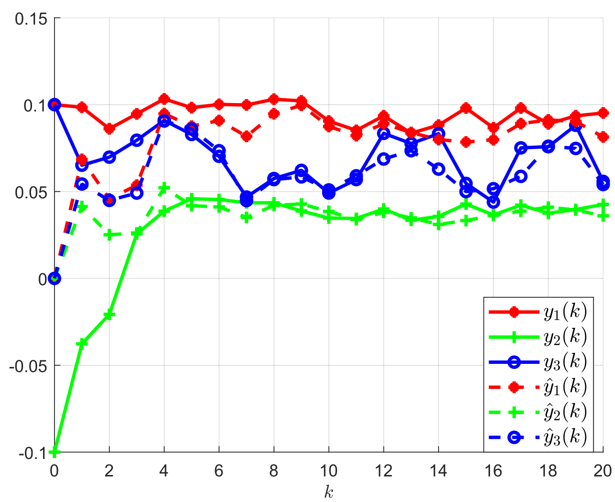

Fig. 3. Trajectories of $\boldsymbol{y}(k)$ and its estimation.

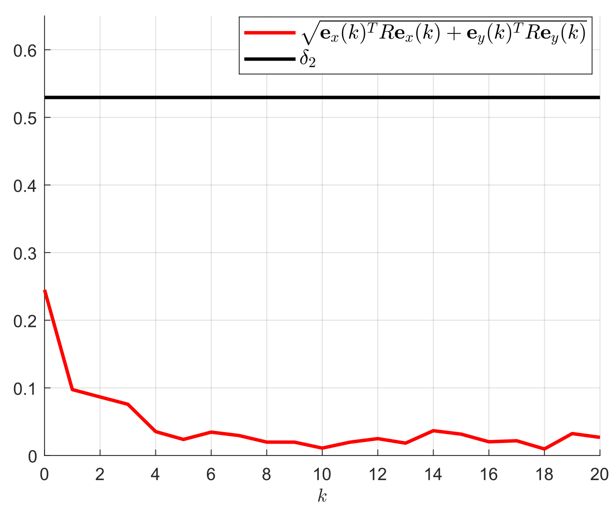

Fig. 4. Trajectories of $(\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k) + \boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k)))^{1/2}$ and δ_2.

$\boldsymbol{y}(k)$ and their estimations are shown in Figs. 2 and 3, respectively. The trajectories of $(\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k) + \boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k))^{1/2}$ and δ_2 are shown in Fig. 4. From Fig. 4, the value of $(\boldsymbol{e}_x(k)^T R\boldsymbol{e}_x(k) + \boldsymbol{e}_y(k)^T R\boldsymbol{e}_y(k)))^{1/2} \leqslant \delta_2 = 0.5294$, which means that the condition (13) in Definition 1 holds, that is, the EES (12) satisfies the SFTB.

TABLE I

BOUND δ_2^2 FOR DIFFERENT NONFRAGILE ESTIMATORS UNDER THE SAME SYSTEM PARAMETERS

γ	l_0	N	δ_1^2	PU model δ_2^2	IU model δ_2^2
1.02	1.5	20	0.06	0.2803	0.2812
1.02	1.5	20	0.08	0.3737	0.3750
1.02	1.5	21	0.08	0.3812	0.3825
1.02	5.0	21	0.08	0.3820	0.3832
1.02	5.0	20	0.08	0.3745	0.3757
1.02	0.1	20	0.08	0.3734	0.3747
1.01	1.5	20	0.08	0.3117	0.3127
1.03	1.5	20	0.08	0.4474	0.4489

Remark 3: In order to compare the PU model with the IU model for the estimator, the estimator gains are described by the IU model as

$$K_x \triangleq K_1 + H_1\Lambda_x(k)H_2, \quad K_y \triangleq K_2 + H_3\Lambda_y(k)H_4$$

where K_1 and K_2 are the controller gains. $H_1\Lambda_x(k)H_2$ and $H_3\Lambda_y(k)H_4$ are uncertain, where H_1, H_2, H_3, and H_4 are constant matrices with proper dimension, and the time-varying matrices $\Lambda_x(k)$ and $\Lambda_y(k)$ satisfy $\Lambda_x(k)^T\Lambda_x(k) \preceq I$ and $\Lambda_y(k)^T\Lambda_y(k) \preceq I$, respectively. From Table I, it is observed that the PU model is less conservative than the IU model for the estimator gains by comparative experiments under the same system parameters.

Remark 4: From the process of the simulation, the system matrices $A_{\delta(k)}$, $B_{\delta(k)}$, $C_{\delta(k)}$, $D_{\delta(k)}$, $E_{\delta(k)}$, and $F_{\delta(k)}$ and the upper bound $\boldsymbol{\ell}_1$ of the nonlinear function $\boldsymbol{g}(\cdot)$ are the main parameters that we tried many times to debug with different data to ensure the existence of solution. Through the simulation, one of our results validates that the nonlinear constraint $\boldsymbol{\ell}_1$ needs to be limited. According to (36), the performance of the proposed method is mainly affected by the TVDs parameters $d_{\delta(k)}$ and $\tau_{\delta(k)}$, the initial bound δ_1^2 of system, the energy bound of noises c^2, the maximum interval N of the finite time, and the scalars l_0 and γ. It follows from Table I that the bound δ_2^2 increases with the aforementioned factors N, l_0, γ, and δ_1^2 rising under given delays conditions.

V. CONCLUSION

In this article, the finite-time state estimation for Markovian BAM NNs with asymmetrical SMD TVDs and inconstant measurements has been investigated. A new TVDs mode where the time-delay interval was SMD has been proposed. A more general measurement mode has been adopted, which was a measurement environment-dependent Markov chain and the transition probability was SMD. A PU model has been used to reduce the conservative property and improve the robustness of the estimator. By constructing a new Lyapunov–Krasovskii functional function, sufficient conditions have been derived to ensure that the EES was SFTB. A simulation result has been provided to illustrate the effectiveness of the method. In summary, this article has proposed a more general model of BAM NNs, and a more detailed delay model has been analyzed, which can be easily applied to other physical models.

REFERENCES

[1] A. Dundar, J. Jin, B. Martini, and E. Culurciello, "Embedded streaming deep neural networks accelerator with applications," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 7, pp. 1572–1583, Jul. 2017.

[2] B. Xu, Q. Liu, and T. Huang, "A discrete-time projection neural network for sparse signal reconstruction with application to face recognition," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 1, pp. 151–162, Jan. 2019.

[3] M. Moalla, H. Frigui, A. Karem, and A. Bouzid, "Application of convolutional and recurrent neural networks for buried threat detection using ground penetrating radar data," *IEEE Trans. Geosci. Remote Sens.*, vol. 58, no. 10, pp. 7022–7034, Oct. 2020.

[4] L. Liu, Y.-J. Liu, and S. Tong, "Neural networks-based adaptive finite-time fault-tolerant control for a class of strict-feedback switched nonlinear systems," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2536–2545, Jul. 2019.

[5] B. Kosko, "Bidirectional associative memories," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-18, no. 1, pp. 49–60, Jan./Feb. 1988.

[6] J. Cao and Y. Wan, "Matrix measure strategies for stability and synchronization of inertial BAM neural network with time delays," *Neural Netw.*, vol. 53, pp. 165–172, May 2014.

[7] X. Yang, Z. Feng, J. Feng, and J. Cao, "Synchronization of discrete-time neural networks with delays and Markov jump topologies based on tracker information," *Neural Netw.*, vol. 85, pp. 157–164, Jan. 2017.

[8] X. Yang, Q. Song, J. Cao, and J. Lu, "Synchronization of coupled Markovian sneural networks with proportional delays via quantized control," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 3, pp. 951–958, Mar. 2019.

[9] J. Wang, C. Yang, H. Shen, J. Cao, and L. Rutkowski, "Sliding-mode control for slow-sampling singularly perturbed systems subject to Markov jump parameters," *IEEE Trans. Syst., Man, Cybern. Syst.*, pp. 1–8, 2020, doi: 10.1109/TSMC.2020.2979860.

[10] J. Wang, T. Ru, H. Shen, J. Cao, and J. H. Park, "Finite-time $\mathcal{L}_2$-$\mathcal{L}_\infty$ synchronization for semi-Markov jump inertial neural networks using sampled data," *IEEE Trans. Netw. Sci. Eng.*, vol. 8, no. 1, pp. 163–173, 2021.

[11] J. Wang, J. Xia, H. Shen, M. Xing, and J. H. Park, "H_∞ synchronization for fuzzy Markov jump chaotic systems with piecewise-constant transition probabilities subject to pdt switching rule," *IEEE Trans. Fuzzy Syst.*, early access, Sep. 29, 2020, doi: 10.1109/TFUZZ.2020.3012761.

[12] H. Shen, J.-C. Wu, J.-W. Xia, and Z. Wang, "$\mathcal{H}_\infty$ state estimation for Markov jump neural networks with transition probabilities subject to the persistent dwell-time switching rule," *Chin. Phys. B*, vol. 30, no. 6, May 2021, Art. no. 060203.

[13] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays," *IEEE Trans. Neural Netw.*, vol. 22, no. 10, pp. 1566–1575, Oct. 2011.

[14] C. Sowmiya, R. Raja, Q. Zhu, and G. Rajchakit, "Further mean-square asymptotic stability of impulsive discrete-time stochastic BAM neural networks with Markovian jumping and multiple time-varying delays," *J. Franklin Inst.*, vol. 356, no. 1, pp. 561–591, Jan. 2019.

[15] M. S. Ali, K. Meenakshi, and Y. H. Joo, "Finite-time H_∞ filtering for discrete-time Markovian jump BAM neural networks with time-varying delays," *Int. J. Control, Autom. Syst.*, vol. 16, no. 4, pp. 1971–1980, Aug. 2018.

[16] K. Wu and J. Jian, "Global robust exponential dissipativity of uncertain second-order BAM neural networks with mixed time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Oct. 20, 2020, doi: 10.1109/TNNLS.2020.3027326.

[17] Y. Liu, Z. Wang, J. Liang, and X. Liu, "Stability and synchronization of discrete-time Markovian jumping neural networks with mixed mode-dependent time delays," *IEEE Trans. Neural Netw.*, vol. 20, no. 7, pp. 1102–1116, Jul. 2009.

[18] Y. Liu, Z. Wang, and X. Liu, "State estimation for discrete-time Markovian jumping neural networks with mixed mode-dependent delays," *Phys. Lett. A*, vol. 372, no. 48, pp. 7147–7155, Dec. 2008.

[19] E. K. Boukas and Z. K. Liu, "Robust H_∞ control of discrete-time Markovian jump linear systems with mode-dependent time-delays," *IEEE Trans. Autom. Control*, vol. 46, no. 12, pp. 1918–1924, Jun. 2001.

[20] S. Yan, M. Shen, S. K. Nguang, and G. Zhang, "Event-triggered H_∞ control of networked control systems with distributed transmission delay," *IEEE Trans. Autom. Control*, vol. 65, no. 10, pp. 4295–4301, Nov. 2019.

[21] H. Rao, L. Zhao, Y. Xu, Z. Huang, and R. Lu, "Quasisynchronization for neural networks with partial constrained state information via intermittent control approach," *IEEE Trans. Cybern.*, early access, Mar. 11, 2021, doi: 10.1109/TCYB.2021.3049638.

[22] Y. Xu, Z. Huang, H. Rao, R. Lu, and T. Huang, "Quasi-synchronization for periodic neural networks with asynchronous target and constrained information," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 7, pp. 4379–4388, Jul. 2021, doi: 10.1109/TSMC.2019.2930971.

[23] Z. Du, Y. Kao, K. H. Reza, and X. Zhao, "Interval type-2 fuzzy sampled-data H_∞ control for nonlinear unreliable networked control systems," *IEEE Trans. Autom. Control*, vol. 28, no. 7, pp. 1434–1448, 2020.

[24] J. Chen, S. Kher, and A. Somani, "Distributed fault detection of wireless sensor networks," in *Proc. Workshop Dependability Issues Wireless AD Hoc Netw. Sensor Netw.*, 2006, pp. 65–72.

[25] J. Zhang, S. Gao, G. Li, J. Xia, X. Qi, and B. Gao, "Distributed recursive filtering for multi-sensor networked systems with multi-step sensor delays, missing measurements and correlated noise," *Signal Process.*, vol. 181, Apr. 2021, Art. no. 107868.

[26] L. Yan, C. Di, Q. M. J. Wu, and Y. Xia, "Sequential fusion for multirate multisensor systems with heavy-tailed noises and unreliable measurements," *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, Jul. 7, 2020, doi: 10.1109/TSMC.2020.3003645.

[27] F. Yang, Q.-L. Han, and Y. Liu, "Distributed H_∞ state estimation over a filtering network with time-varying and switching topology and partial information exchange," *IEEE Trans. Cybern.*, vol. 49, no. 3, pp. 870–882, Jan. 2019.

[28] J. Yang, W.-A. Zhang, and F. Guo, "Dynamic state estimation for power networks by distributed unscented information filter," *IEEE Trans. Smart Grid*, vol. 11, no. 3, pp. 2162–2171, May 2020.

[29] W. Yang, C. Yang, H. Shi, L. Shi, and G. Chen, "Stochastic link activation for distributed filtering under sensor power constraint," *Automatica*, vol. 75, pp. 109–118, Jan. 2017.

[30] Y. Xu, J. Zhou, H. Rao, R. Lu, and L. Xie, "Reset moving horizon estimation for quantized discrete time systems," *IEEE Trans. Autom. Control*, early access, Nov. 10, 2020, doi: 10.1109/TAC.2020.3037140.

[31] H. Rao, Y. Guo, Y. Xu, C. Liu, and R. Lu, "Nonfragile finite-time synchronization for coupled neural networks with impulsive approach," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 11, pp. 4980–4989, Nov. 2020.

[32] X. Liu, J. Xia, J. Wang, and H. Shen, "Interval type-2 fuzzy passive filtering for nonlinear singularly perturbed PDT-switched systems and its application," *J. Syst. Sci. Complex.*, Jan. 2021. [Online]. Available: https://link.springer.com/article/10.1007/s11424-020-0106-9, doi: 10.1007/s11424-020-0106-9.

[33] J. Tao, Z.-G. Wu, H. Su, Y. Wu, and D. Zhang, "Asynchronous and resilient filtering for Markovian jump neural networks subject to extended dissipativity," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2504–2513, Jul. 2019.

[34] Y. Xu, L. Yang, Z. Wang, H. Rao, and R. Lu, "State estimation for networked systems with Markov driven transmission and buffer constraint," *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, Mar. 27, 2020, doi: 10.1109/TSMC.2020.2980425.

[35] S. Fan, H. Yan, H. Zhang, H. Shen, and K. Shi, "Dynamic event-based non-fragile dissipative state estimation for quantized complex networks with fading measurements and its application," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 68, no. 2, pp. 856–867, Feb. 2021.

[36] S. Li, X. Wang, and L. Zhang, "Finite-time output feedback tracking control for autonomous underwater vehicles," *IEEE J. Ocean. Eng.*, vol. 40, no. 3, pp. 727–751, Jul. 2015.

[37] X. Yang and J. Lu, "Finite-time synchronization of coupled networks with Markovian topology and impulsive effects," *IEEE Trans. Autom. Control*, vol. 61, no. 8, pp. 2256–2261, Aug. 2016.

[38] W. Qi, G. Zong, and H. R. Karimi, "Finite-time observer-based sliding mode control for quantized semi-Markov switching systems with application," *IEEE Trans. Ind. Informat.*, vol. 16, no. 2, pp. 1259–1271, Feb. 2020.

[39] W.-J. Lin, Y. He, C.-K. Zhang, and M. Wu, "Stochastic finite-time H_∞ state estimation for discrete-time semi-Markovian jump neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 12, pp. 5456–5467, Dec. 2020.

[40] H. Du, G. Wen, Y. Cheng, and J. Lu, "Design and implementation of bounded finite-time control algorithm for speed regulation of permanent magnet synchronous motor," *IEEE Trans. Ind. Electron.*, vol. 68, no. 3, pp. 2417–2426, Mar. 2021.

[41] X. Wan, Z. Wang, Q.-L. Han, and M. Wu, "Finite-time H_∞ state estimation for discrete time-delayed genetic regulatory networks under stochastic communication protocols," *IEEE Trans. Nanobiosci.*, vol. 65, no. 10, pp. 3481–3491, Mar. 2018.

Chang Liu was born in Xiangcheng, China, in 1992. He received the B.S. degree from Henan University, Kaifeng, China, in 2016, and the M.S. degree in control science and engineering with the Guangdong University of Technology, Guangzhou, China, in 2019, where he is currently pursuing the Ph.D. degree in control science and engineering.

His current research interests include networked control systems and finite-time control and estimation.

Zhuo Wang (Member, IEEE) was born in Handan, Hebei, China, in 1983. He received the B.E. degree in automation from Beihang University, Beijing, China, in 2006, and the Ph.D. degree in electrical and computer engineering from the University of Illinois at Chicago, Chicago, IL, USA, in 2013.

He was a Post-Doctoral Fellow with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada, from 2013 to 2014. He worked as a Research Assistant Professor with the Fok Ying Tung Graduate School, The Hong Kong University of Science and Technology, Hong Kong, from 2014 to 2015. He is currently a Professor and a Ph.D. Instructor with the Research Institute for Frontier Science, the Key Laboratory of Ministry of Industry and Information Technology on Quantum Sensing Technology, and the Beijing Advanced Innovation Center for Big Data-Based Precision Medicine, Beihang University, Beijing, China. His research interests include data-based system identification, modeling, analysis, learning, and control; nonlinear system adaptive control; atomic spin ensembles system control; and state estimation and signal processing of complex high-precision measurement instruments.

Dr. Wang is also a Vice Director of the 9th Chinese Association of Automation (CAA) Youth Work Committee, a member of the Adaptive Dynamic Programming and Reinforcement Learning Technical Committee of the IEEE Computational Intelligence Society, the Data Driven Control, Learning and Optimization Professional Committee of CAA, and the Fault Diagnosis and Safety for Technical Processes Professional Committee of CAA. He is also an Associate Editor of IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, *Control Theory and Applications*, *Acta Automatica Sinica*, *Pattern Recognition*, and *Artificial Intelligence*.

Renquan Lu (Senior Member, IEEE) received the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, China, in 2004.

He was supported by the National Science Fund for Distinguished Young Scientists of China in 2014, honored as the Distinguished Professor of Pearl River Scholars Program of Guangdong Province and the Distinguished Professor of Yangtze River Scholars Program by the Ministry of Education of China in 2015 and 2017, respectively. He is currently a Professor with the School of Automation, Guangdong University of Technology, Guangzhou, China. His research interests include complex systems, networked control systems, and nonlinear systems.

Tingwen Huang (Fellow, IEEE) received the B.S. degree from Southwest Normal University (now Southwest University), Chongqing, China, in 1990, the M.S. degree from Sichuan University, Chengdu, China, in 1993, and the Ph.D. degree from Texas A&M University, College Station, TX, USA, in 2002.

After graduation, he worked as a Visiting Assistant Professor with Texas A&M University. In August 2003, he joined Texas A&M University at Qatar (TAMUQ), Doha, Qatar, as an Assistant Professor, where he was promoted to Professor in 2013. He is currently a Professor at TAMUQ. His focus areas for research interests include neural networks, computational intelligence, chaotic dynamical systems, complex networks, and optimization and control.

Yong Xu (Member, IEEE) was born in Zhejiang, China, in 1983. He received the B.S. degree in information engineering from Nanchang Hangkong University, Nanchang, China, in 2007, the M.S. degree in control science and engineering from Hangzhou Dianzi University, Hangzhou, China, in 2010, and the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, in 2014.

He was a Visiting Internship Student with the Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology, Hong Kong, China, from June 2013 to November 2013, where he was a Research Fellow from February 2018 to August 2018. He is currently a Professor with the School of Automation, Guangdong University of Technology, Guangzhou, China. His research interests include PID control, networked control systems, state estimation, and positive systems.