Antenna Theory and Design

Associate Professor
WANG Junjun 王珺珺
School of Electronic and Information Engineering
Beihang University
wangjunjun@buaa.edu.cn
13426405497
New Main Building F1025

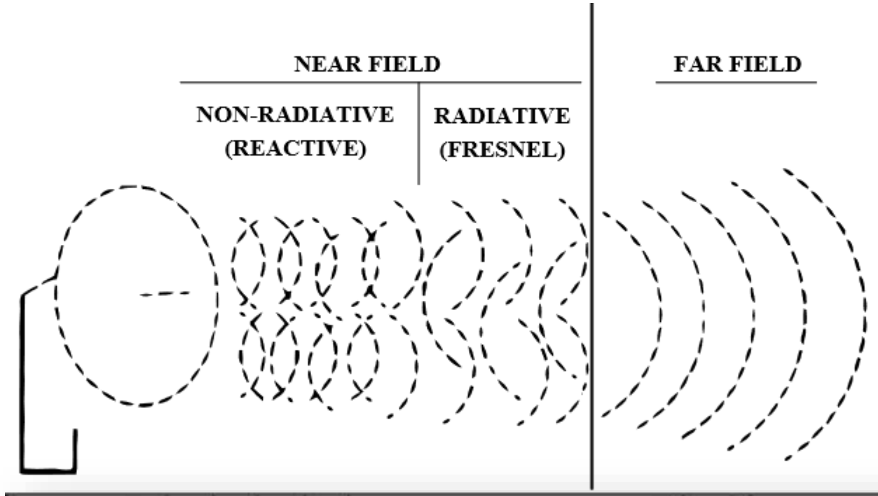
Chapter 3
Point source and array of point sources

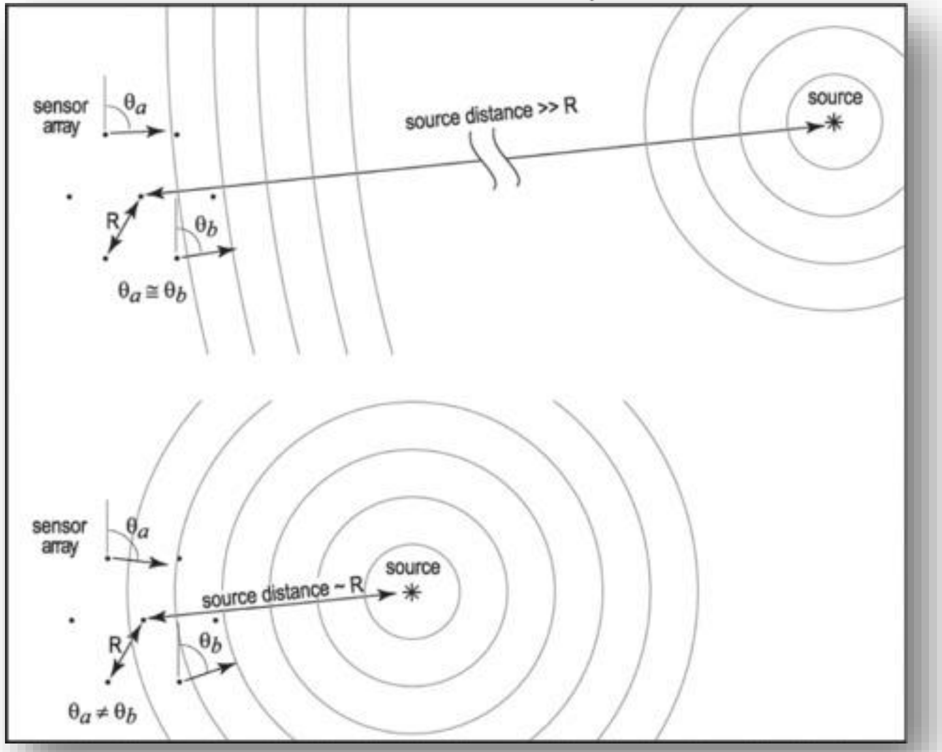
Chapter 3 Point source and arrays of point sources

- Introduction
- Power patterns and examples
- A power theorem
- Array of two isotropic point sources
- Non-isotropic but similar point sources
  and the principle of pattern multiplication

1. Introduction--Point source defined

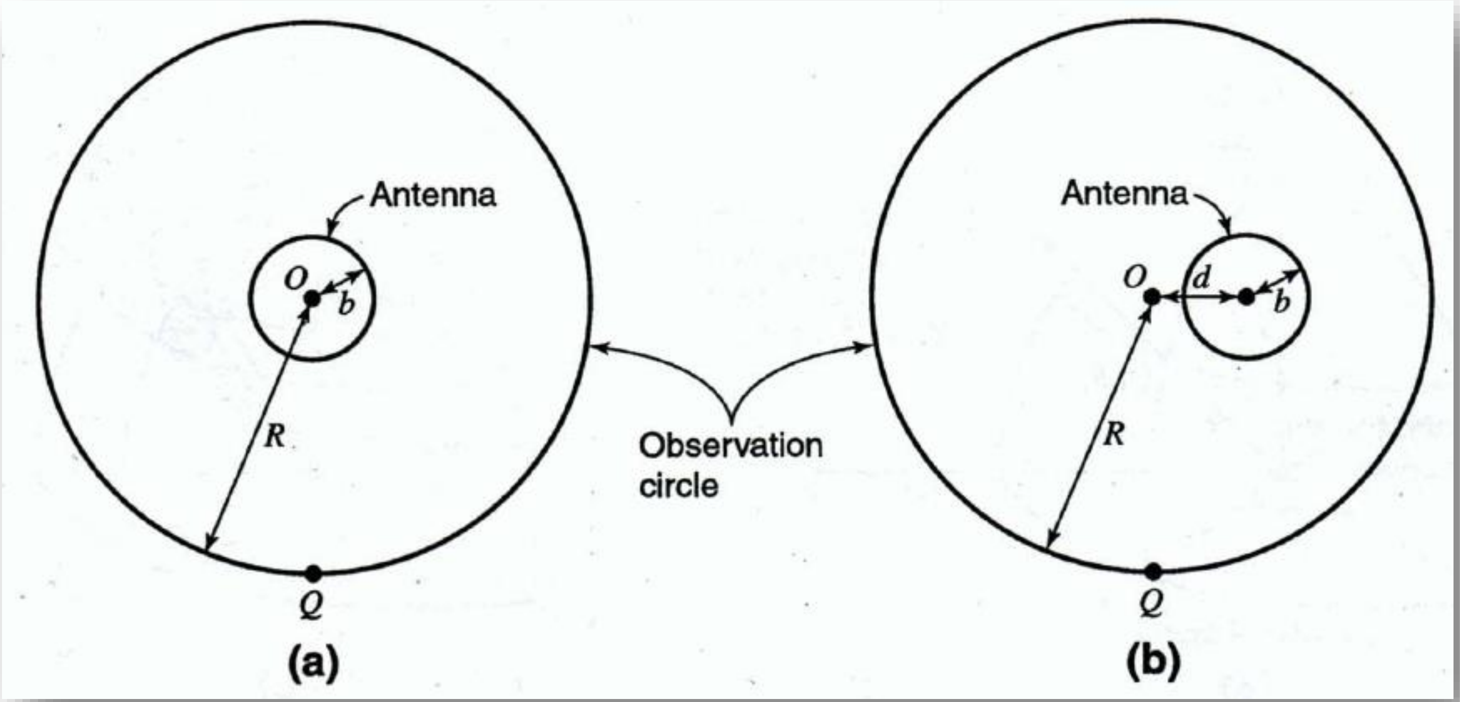
- At a sufficient distance in the far field of an antenna, the radiated fields of the antenna are transverse and the power flow or Poynting vector ($W\ m^{-2}$) is radial as at the point o at a distance R on the observation circle in the figure.
- It is convenient in many analyses to assume that the fields of the antenna are everywhere of this type.
- In fact, we may assume, by extrapolating inward along the radii of the circle, that the waves originate at a fictitious volumeless emitter, or *point source*, at the center o of the observation circle.
- The actual field variation near the antenna, or "near field", is ignored, and we describe the source of the waves only in terms of the "far field" it produces. Provided that our observations are made at a sufficient distance, any antenna, regardless of its size or complexity, can be represented in this way by a single point source.





1. Introduction. Point source defined

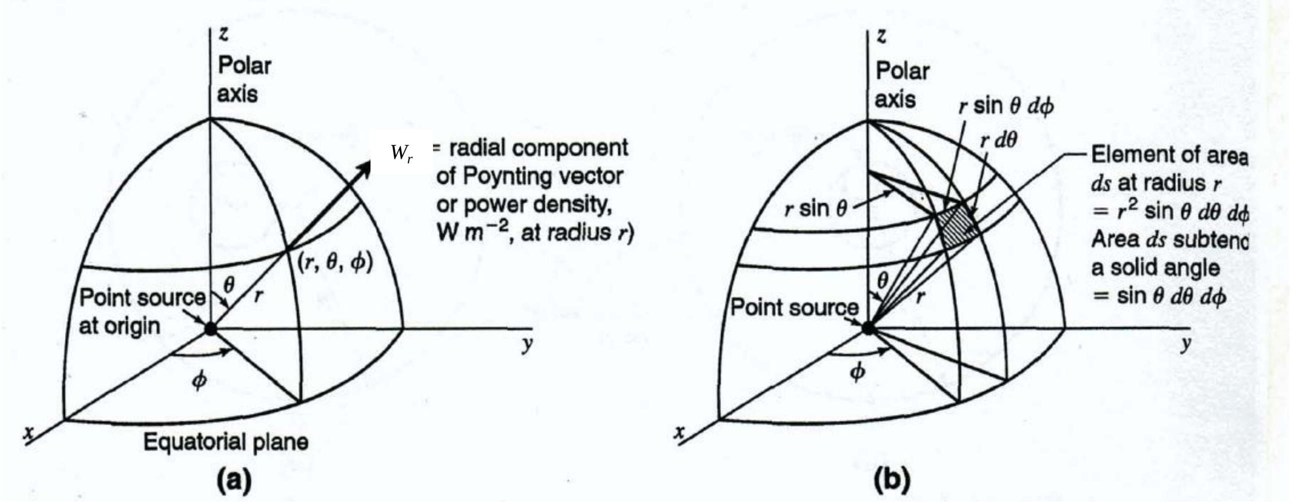
- In Fig. (a), the center *O* of the antenna coincides with the center of the observation circle. If the center of the antenna is displaced from *O*, even to the extent that *O* lies outside the antenna as in Fig. (b), the distance *d* between the two centers has a negligible effect on the field patterns at the observation circle, provided $R >> d$, $R >> b$, and $R >> \lambda$.



- However, the phase patterns will generally differ, depending on *d*. If $d = 0$, the phase shift around the observation circle is usually a minimum. As *d* is increased, the observed phase shift becomes larger.
- Although the cases considered as examples in this chapter are hypothetical, they could be approximated by actual antennas.

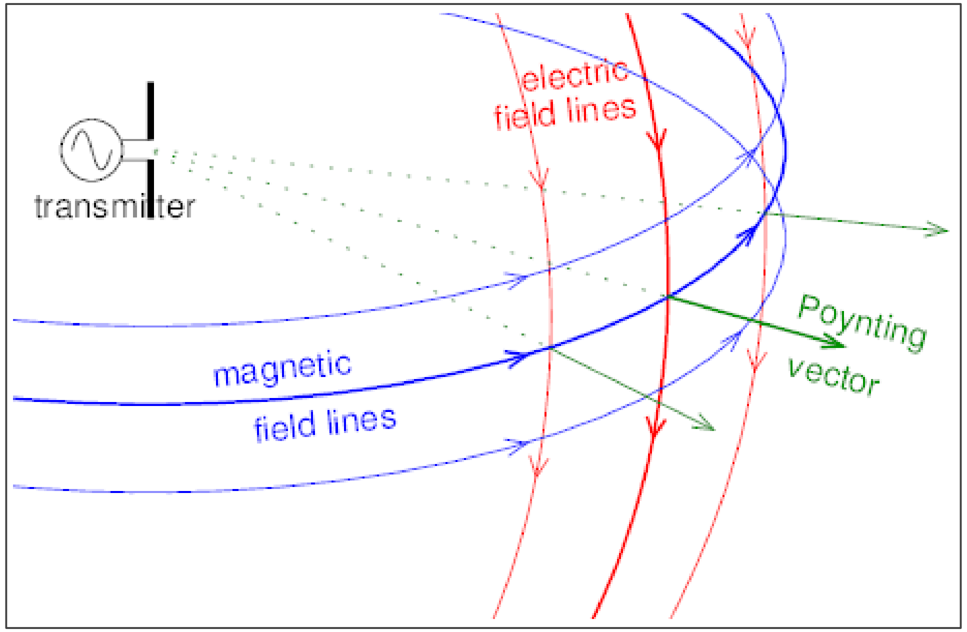
2.Power patterns

- Let a transmitting antenna in free space be represented by a point-source radiator located at the origin of the coordinate as shown in Fig. (a) and (b).

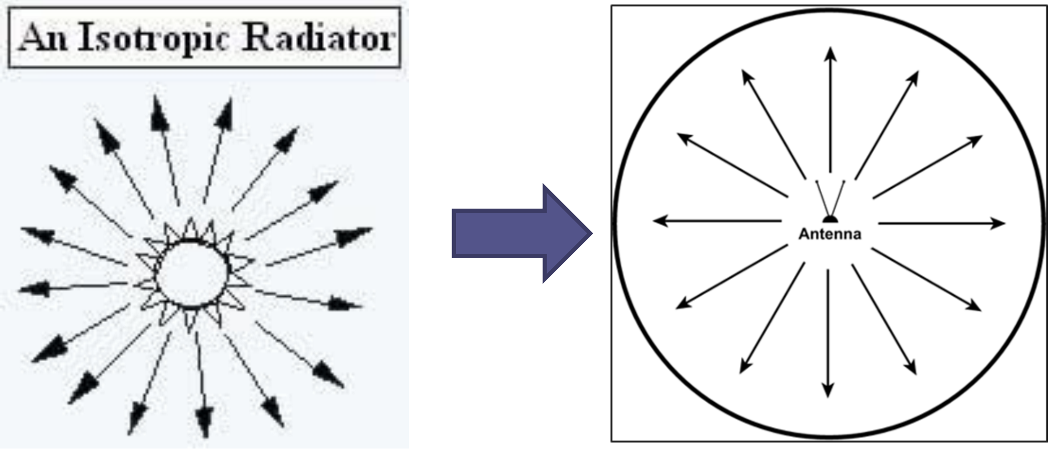


Spherical coordinates for a point source of radiation in free space

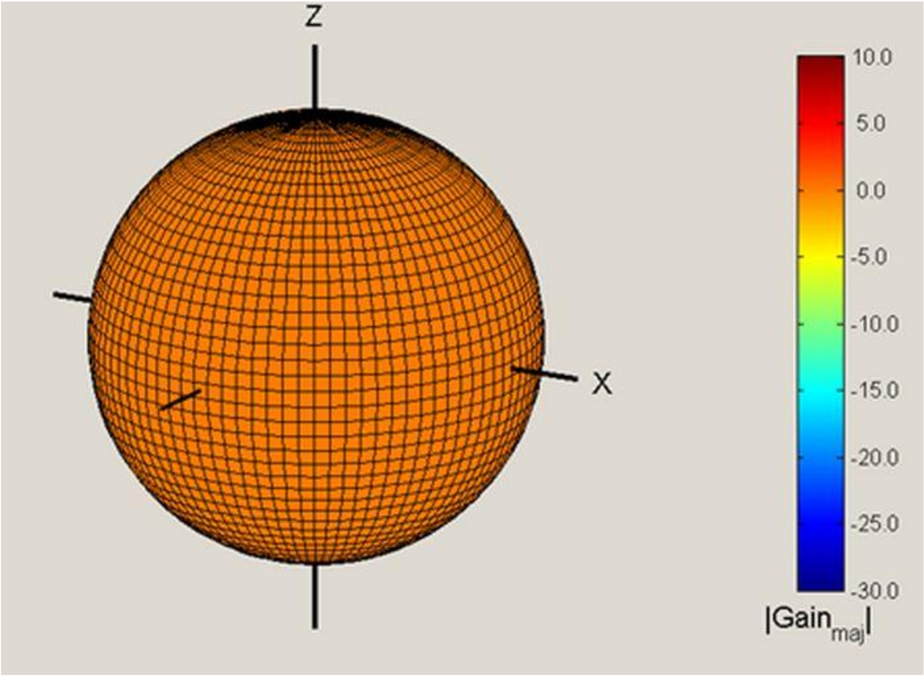
- The radiated energy streams from the source in radial lines. The time rate of energy flow per unit area is the Poynting vector or power density (watts per square meter). For a point source (or in the far field of any antenna), the Poynting vector *W* has only a radial component W_r with no components in either the θ or φ directions ($W_\theta = W_\varphi = 0$). Thus, the magnitude of the Poynting vector, or power density, is equal to the radial component ($|W| = W_r$).



2 Power patterns

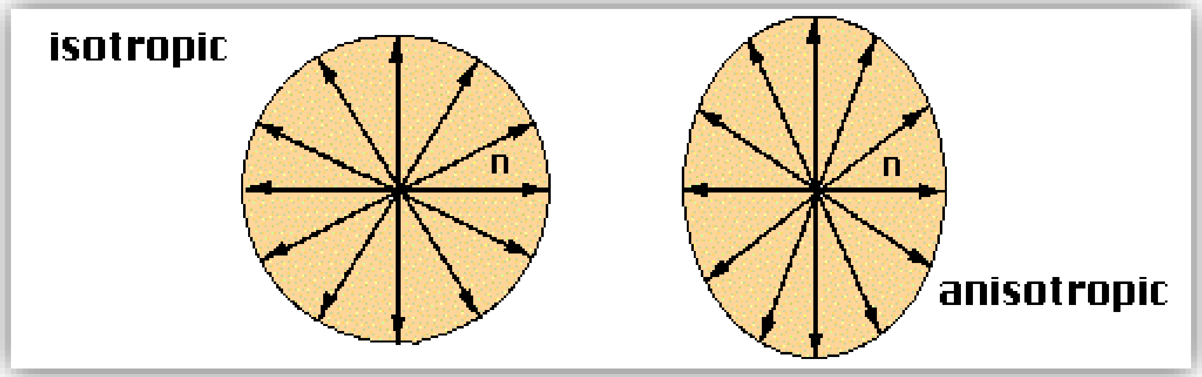


- The three-dimensional power pattern for an isotropic source is a sphere. In two dimensions the pattern is a circle (a cross section through the sphere).

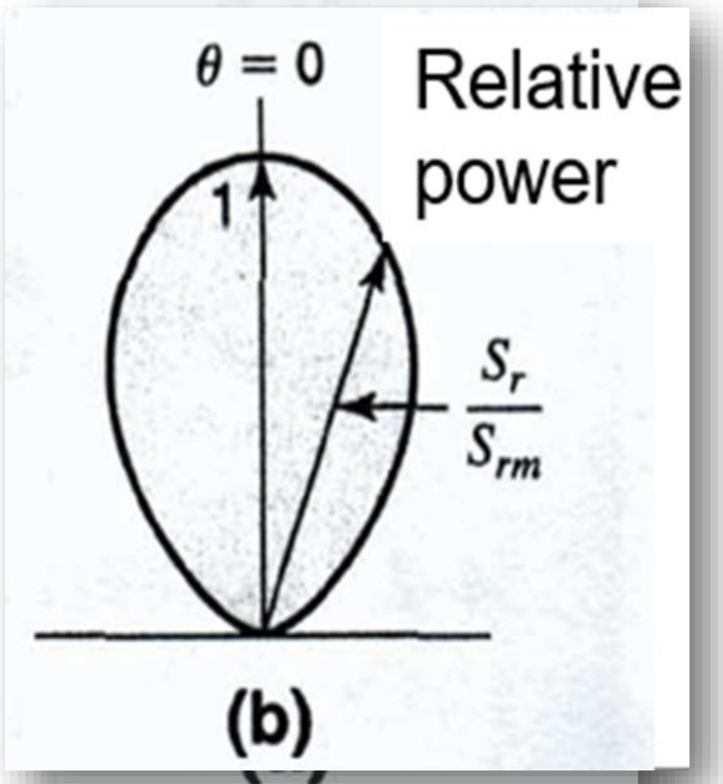


- A source that radiates energy uniformly in all directions is an isotropic source.
- For such a source the radial component W_r of the Poynting vector is independent of θ and φ. A graph of W_r at a constant radius as a function of angle is a Poynting vector, or power-density, pattern, but is usually called a *power pattern*.

2.Power patterns



- Although the isotropic source is convenient in theory, it is not a physically realizable type. Even the simplest antennas have directional properties, i.e., they radiate more energy in some directions than in others. In contrast to the isotropic source, they might be called *anisotropic* sources.



- They can be expressed in absolute power pattern (expressed in watts per square meter) or relative power pattern (expressed in terms of its value in some reference direction).
- It is customary to take the reference direction such that W_r is a maximum. Thus, the radius for relative power is W_r/W_{rm}, where W_{rm} is the maximum value of W_r. The maximum value of the relative power pattern is unity.
- A pattern with a maximum of unity is also called a normalized pattern.

3. A power theorem and its application to an isotropic source

- If the Poynting vector is known at all points on a sphere of radius *r* from a point source in a lossless medium, the *total-power radiated by the source is the integral over the surface of the sphere of the radial component* W_r *of the average Poynting vector.*
- Thus

$$\mathrm{P}= \oiint W ds = \oiint W_r \, ds$$

- Where *P* = power radiated, *W*
- W_r = radial component of average Poynting vector, $\mathrm{W/m^{-2}}$
- *ds* = infinitesimal element of area of sphere= = $r^2 \sin\theta \, d\theta \, d\varphi$, m^2.

3.A power theorem and its application to an isotropic source

- For an isotropic source, W_r is independent of θ and φ,

$$P = \oiint W ds = \oiint W_r \, ds$$

- Which indicates that the magnitude of the *Poynting vector varies inversely as the square of the distance* from a point-source radiator. This is a statement of the well-known law for the variation of power per unit area as a function of the distance.

$$P = W_r \oiint ds = W_r \times 4\pi r^2$$

$$W_r = \frac{P}{4\pi r^2}$$

3. A power theorem and its application to an isotropic source

Radiation intensity

- The radiation intensity U is expressed in watts per unit solid angle. The radiation intensity is independent of radius. It is power per steradian.

$$r^2 W_r = P/4\pi = U$$

- Thus, the *power theorem* may be restated as follows:
  - The total power radiated is given by the integral of the radiation intensity over a solid angle of 4π steradians.
- Applying the equation to an isotropic source gives

$$P = 4\pi U_0$$

- where U_o = radiation intensity of isotropic source

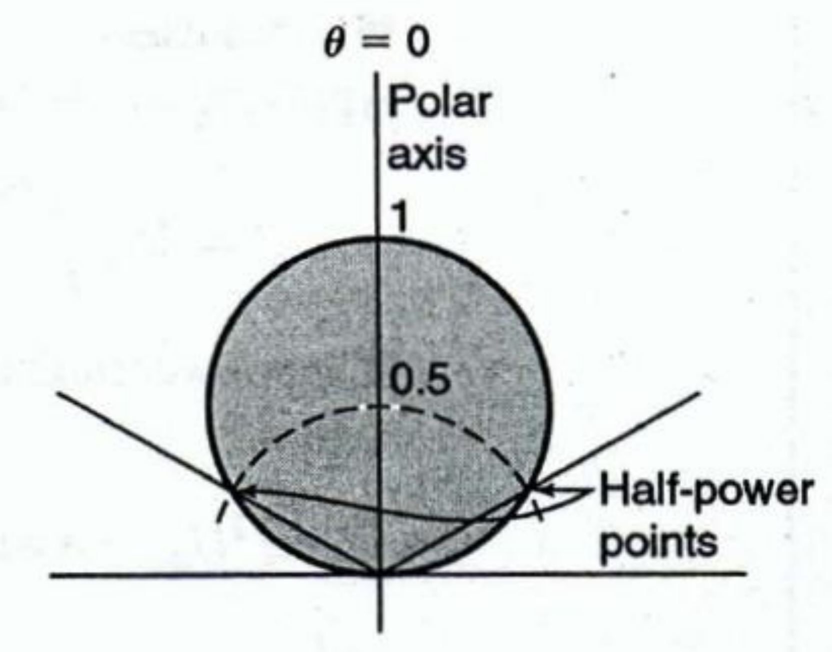
Examples of power patterns

① Source with Unidirectional Cosine Power Pattern

- A source has a cosine radiation-intensity pattern, that is,

  $U = U_m \cos \theta$, where U_m = maximum radiation intensity.

  The radiation intensity U has a value only in the upper hemisphere ($0 \leq \theta \leq \pi/2$ and $0 \leq \varphi \leq 2\pi$) and is zero in the lower hemisphere. The radiation intensity is a maximum at $\theta = 0$. The space pattern is a figure of revolution of this circle around the polar axis. Find the directivity.



Unidirectional cosine power pattern

Examples of power patterns

① Source with Unidirectional Cosine Power Pattern

- *Answer:*
- To find the total power radiated by the cosine source, we integrate only over the upper hemisphere. Thus

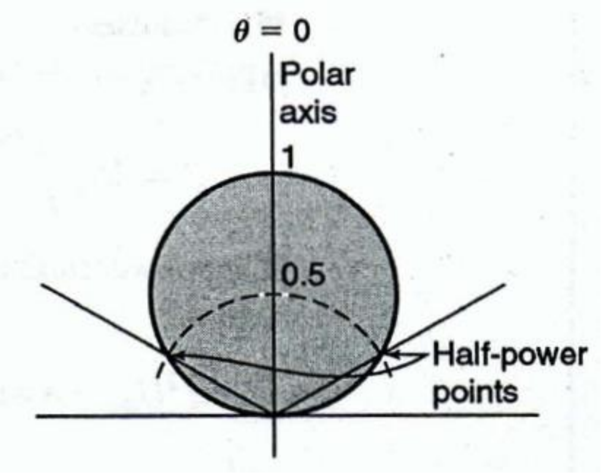
$$P = \int_{U}^{2\pi} \int_{0}^{\frac{\pi}{2}} U_m \cos\theta \sin\theta \, d\theta \, d\phi = \pi U_m$$

- If the power radiated by the unidirectional cosine source is the same as for an isotropic source, then

$$\pi U_m = 4\pi U_0$$

- Or

$$Directivity = \frac{U_m}{U_0} = 4 = D \qquad Ans.$$



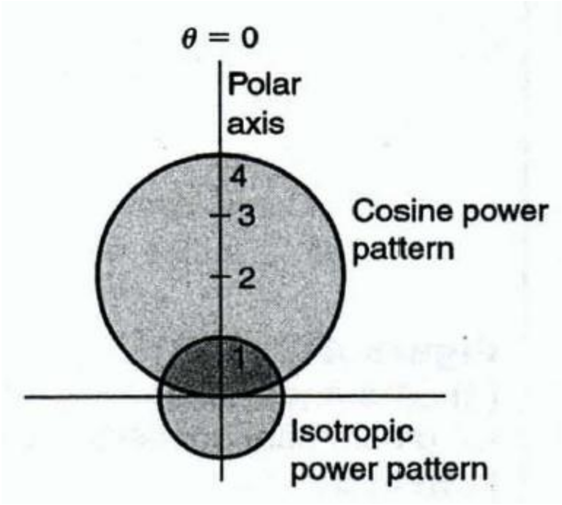
Unidirectional cosine power pattern

Examples of power patterns

A source has a cosine radiation-intensity pattern, that is,

$U = U_m \cos\theta$, where U_m = maximum radiation intensity.

The radiation intensity U has a value only in the upper hemisphere ($0 \le \theta \le \pi/2$ and $0 \le \varphi \le 2\pi$) and is zero in the lower hemisphere. The radiation intensity is a maximum at $\theta = 0$. The space pattern is a figure of revolution of this circle around the polar axis. Find the directivity.



Power patterns of cosine and isotropic sources.

- *Answer:*
- Thus, the maximum radiation intensity U_m of the unidirectional cosine source (in the direction $\theta = 0$) is 4 times the radiation intensity U_o from an isotropic source radiating the same total power. The power patterns for the two sources are compared in the following Figure for the same total power radiated by each.

Examples of power patterns

② Source with Bidirectional Cosine Power Pattern

- A source has a cosine power pattern that is bidirectional. Find the directivity. With radiation in two hemispheres instead of one; the maximum radiation intensity is half its value in example 1. Thus,
  - D=4/2=2 Ans

Examples of power patterns

③ Source with Sine-Squared (Doughnut)Power Pattern

- A source has a sine-squared radiation-intensity power pattern. The radiation-intensity pattern is given by

$$U = U_m \sin^2\theta$$

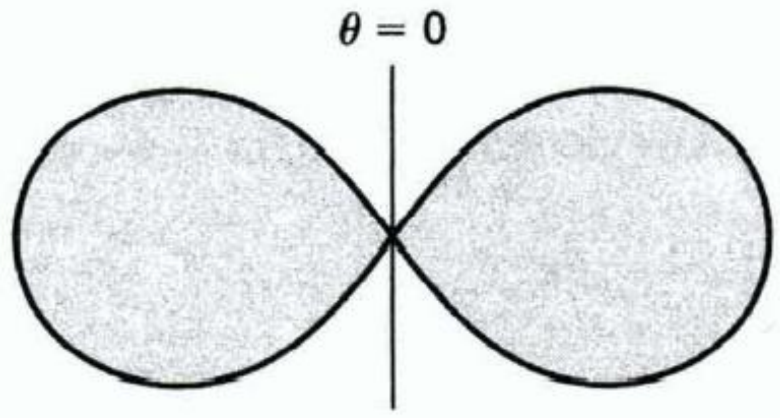
- The power pattern is shown in Fig. a. This type of pattern is of considerable interest because it is the pattern produced by a short dipole coincident with the polar ($\theta = 0$) axis in Fig. a.
  - The total power radiated is

$$P = U_m \int_0^{2\pi}\int_0^{\pi} sin^3\theta \, d\theta \, d\phi = \frac{8}{3}\pi U_m$$

- If P is the same as for the isotropic source,

$$\frac{8}{3}\pi U_m = 4\pi U_0$$

$$Directivity = \frac{U_m}{U_0} = 1.5 = D \qquad Ans.$$



(a)

Examples of power patterns

④ Source with Unidirectional Cosine-Squared Power Pattern

- A source with a unidirectional cosine-squared radiation-intensity power pattern is given by

  $U = U_m \cos^2\theta$

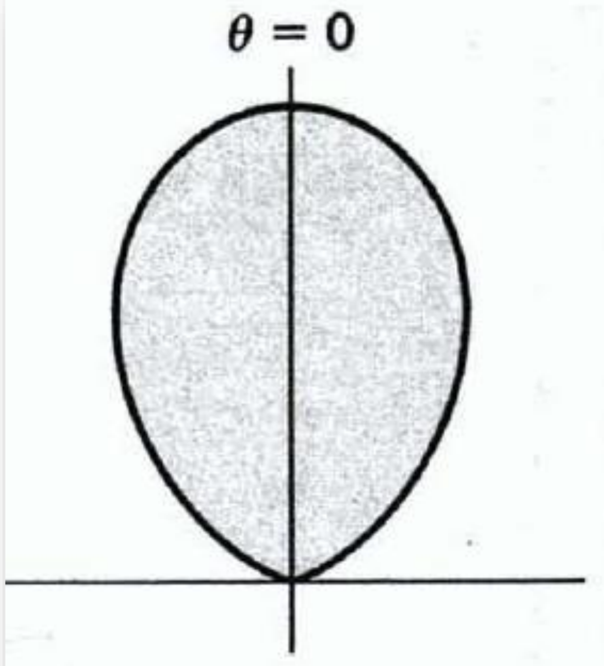
- The radiation intensity has a value only in the upper hemisphere as shown in Fig. b. The 3-D or space pattern is a figure-of-revolution of this pattern around the polar ($\theta = 0$) axis. Find the directivity.
  - The total power radiated is

$$P = U_m \int_0^{2\pi} \int_0^{\pi/2} cos^2\theta sin\theta \, d\theta \, d\phi = \frac{2}{3}\pi U_m$$

- If P is the same as for the isotropic source,

$$\frac{2}{3}\pi U_m = 4\pi U_0$$

$$Directivity = \frac{U_m}{U_0} = 6 = D \qquad Ans.$$



(b)

Examples of power patterns

Directivities of the point source patterns are summarized as below:

pattern	directivity
Unidirectional cosine	4
Bidirectional cosine	2
Sine doughnut	1.27
Sine-squared doughnut	1.5
Unidirectional cosine squared	6

- Actually, some examples can provides some valuable insights into the effect minor lobes have on directivity or gain. *Without minor lobes the gain of this antenna would be 91.4 or 19.6 dBi as compared to a gain of 18.0 or 12.6 dBi with minor lobes.* The minor lobes have large beam or solid angles because they extend 360^0 in the azimuth or ϕ direction at large $sin\theta$ values (θ near 90^0). The main lobe, on the other hand, is at small θ angles so the $P_n(\theta)\ sin\ \theta$ product is small, in fact, zero at $\theta = 0^0$.

4. Arrays of two isotropic point sources

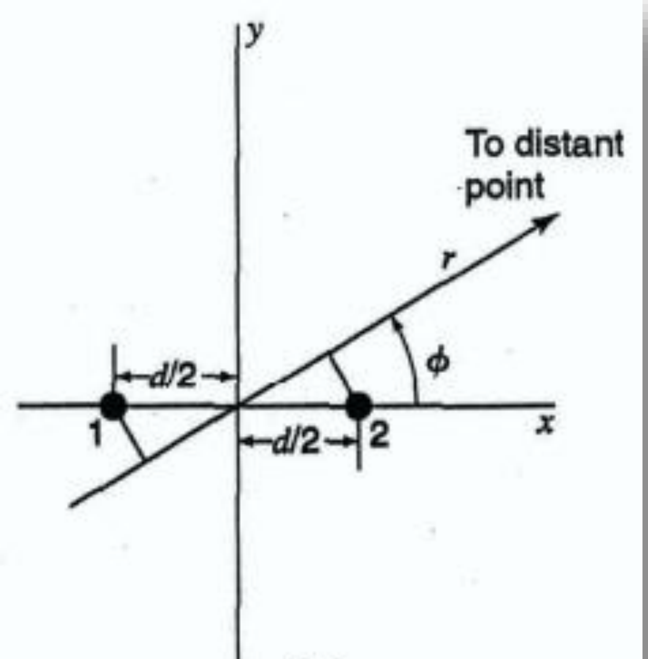
- We continue with the point-source concept, but extend it to a consideration of arrays of point sources. This approach is of great value since ***the pattern of any antenna can be regarded as produced by an array of point sources***. Much of the discussion will concern arrays of isotropic point sources which may represent different kinds of antennas.

- Let us introduce the subject of arrays of point sources by considering the simplest situation, namely, that of two isotropic point sources. As illustrations, two cases involving two isotropic point sources are discussed.

4. Arrays of two isotropic point sources

Case 1. Two Isotropic Point Sources of Same Amplitude and Phase

- Let the two point sources, 1 and 2, be separated by a distance d and located symmetrically with respect to the origin of the coordinates as shown in Fig. a. The angle ϕ is measured counterclockwise from the positive x axis. The origin of the coordinates is taken as the reference for phase.



(a)

- At a distant point in the direction ϕ the field from source 1 is retarded by 1/2 $d_r \cos\phi$, while the field from source 2 is advanced by 1/2 $d_r \cos\phi$, where d_r is the distance between the sources expressed in radians.

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

4. Arrays of two isotropic point sources

- The total field at a large distance r in the direction ϕ is then

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2} \qquad (1)$$

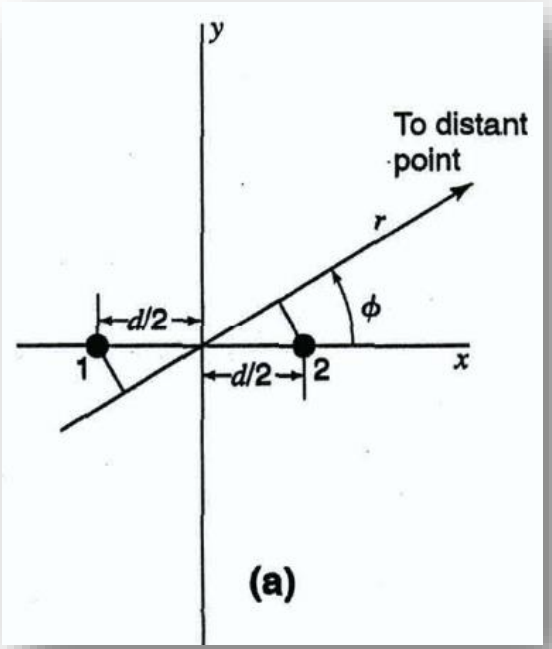
- where $\psi = d_r \cos\phi$ and the amplitude of the field components at the distance r is given by E_o.
  - The first term is the component of the field due to source 1 and the second term is the component due to source 2. Then rewritten as:

$$E = 2E_0 \frac{e^{j\psi/2} + e^{-j\psi/2}}{2} \qquad (2)$$

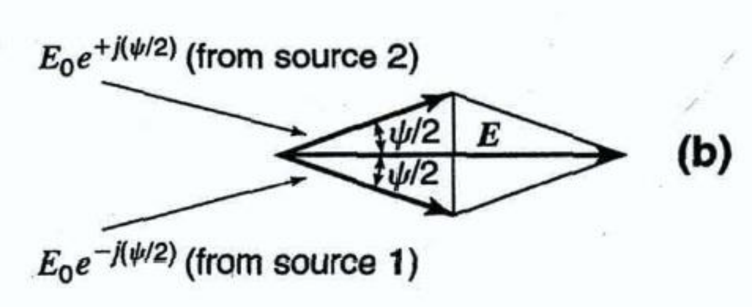
  - which by a trigonometric identity is

$$E = 2E_0 \cos\frac{\psi}{2} = 2E_0 \cos\left(\frac{d_r}{2}\cos\phi\right) \qquad (3)$$

  - This result may also be obtained with the aid of the vector diagram shown in Fig. b, from which the above equation follows directly.



Relation to coordinate system of two isotropic point sources separated by a distance *d*



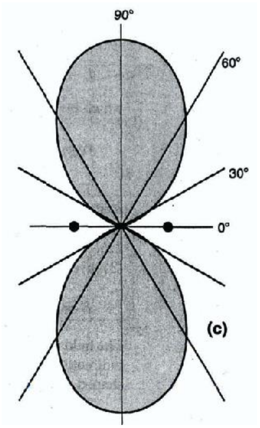
Vector addition of the fields from two isotropic point sources of equal amplitude and same phase located as in (a)

4. Arrays of two isotropic point sources

- We note in Fig. b that the phase of the total field *E* does not change as a function of ψ. To normalize (3), that is, make its maximum value unity, set $2E_o = 1$. Suppose further that *d* is λ/2. Then $d_r = \pi$. Then

$$E = \cos\left(\frac{\pi}{2}\cos\phi\right) \qquad (4)$$

- The field pattern of *E* versus ϕ as expressed by (4) is presented in Fig.c.



Field pattern of two isotropic point sources of equal amplitude and same phase located as in (a) for the case where the separation d=λ/2.

The pattern is a bidirectional figure-of-eight with maxima along the y axis. The space pattern is doughnut-shaped, being a figure-of-revolution of this pattern around the x axis.

4. Arrays of two isotropic point sources

Case 2. Two Isotropic Point Sources of Same Amplitude but Opposite Phase

- This case is identical with the one we have just considered except that the two sources are in opposite phase instead of in the same phase.

- Then the total field in the direction ϕ at a large distance r is given by

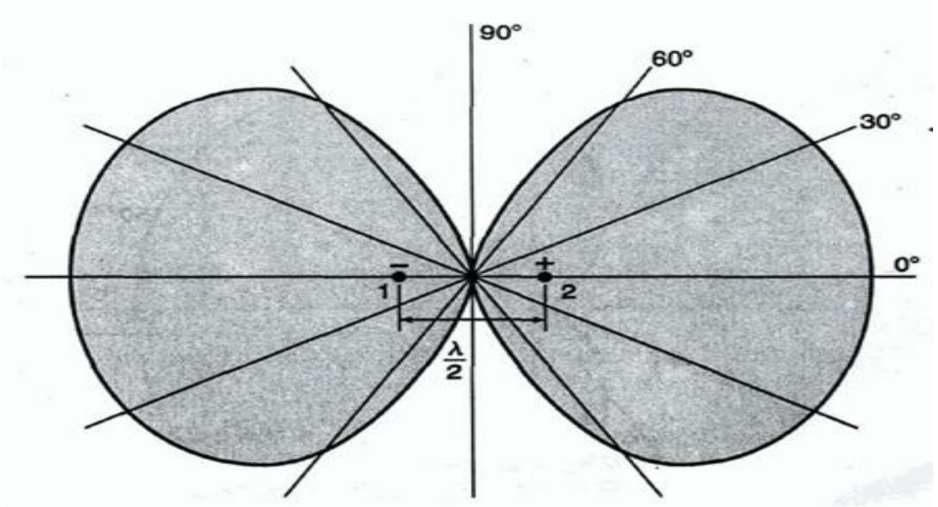
$$E = E_0 e^{-j\psi/2} - E_0 e^{j\psi/2} \tag{5}$$

- From which

- $$E = 2jE_0\, sin\frac{\psi}{2} = 2jE_0\, sin\left(\frac{d_r}{2}\cos\phi\right) \tag{6}$$

4. Arrays of two isotropic point sources

- Whereas in Case 1 (3) involves the cosine of ψ /2, (6) for Case 2 involves the sine. Equation (6) also includes an operator *j*, indicating that the phase reversal of one of the sources in Case 2 results in a 90^0 phase shift of the total field as compared with the total field for Case 1.
- Thus, putting $2jE_o = 1$ and considering the special case of $d = \lambda/2$, (6) becomes
- $$E = sin\left(\frac{\pi}{2}\cos\phi\right) \quad (7)$$
  - The field pattern given by (7) is shown in the following Figure.



The pattern is a relatively broad figure-of-eight with maximum field in the same direction as the line joining the sources (x axis). The space pattern is a figure-of-revolution of this pattern around x axis. The two sources,in this case, may be described as a simple type of "end-fire" array. In contrast to this pattern, the in-phase point sources produce a pattern with the maximum fieldnormal to the line joining the sources. The two sources for this case may be described as a simple "broadside" type of array.

4. Arrays of two isotropic point sources

- Case 3. Two Isotropic Point Sources of the Same Amplitude and In-Phase Quadrature
- Let source 1 be retarded by 45° and source 2 advanced by 45 °.
- Then the total field in the direction ϕ at a large distance r is given by

$$E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{z} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{z} + \frac{\pi}{4}\right)\right] \qquad (8)$$

- From which

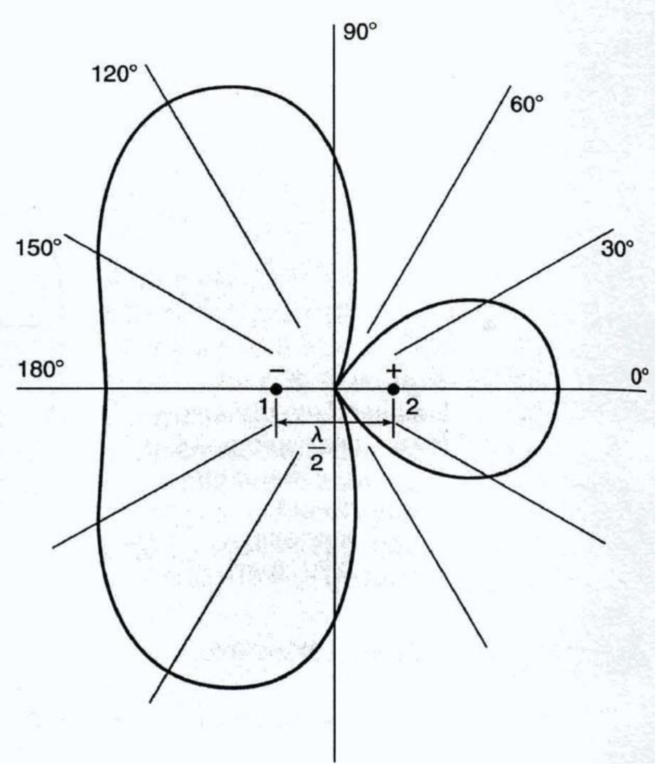
$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right) \qquad (9)$$

- Thus, putting $2E_o = 1$ and considering the special case of $d = \lambda/2$, (9) becomes

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right) \qquad (10)$$

4. Arrays of two isotropic point sources

- The field pattern given by (7) is shown in the following Figure.



Relative field pattern for two isotropic point sources of The same amplitude and in phase quadrature for the spacing of λ/2.

The source to the right leads that to the left by 90°

The space pattern is figure-of-revolution of this pattern around *x* axis. Most of the radiation is in the second and third quadrants. It's interesting to note that the field in the direction ϕ=0° is the same as in the direction ϕ=180°. The directions ϕ_m of maximum field are obtained by setting the argument of (9) equal to kπ.

$$\frac{\pi}{4}+\frac{\pi}{2}\cos\phi_m = k\pi$$

For k=0,

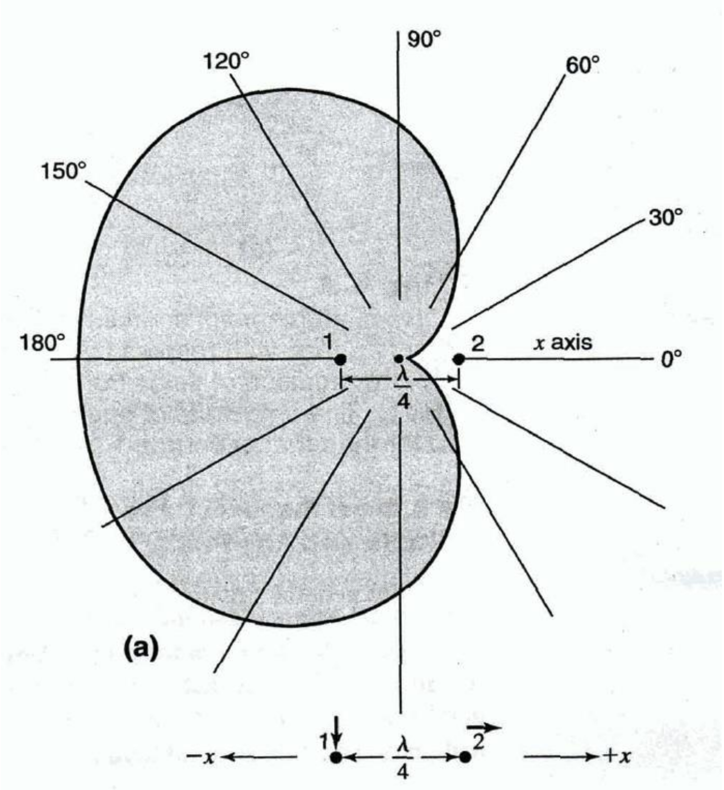
$$\frac{\pi}{2}\cos\phi_m = -\frac{\pi}{4}$$

and

$$\phi_m=120^\circ \text{ and } 240^\circ$$

4. Arrays of two isotropic point sources

- Case 3. Two Isotropic Point Sources of the Same Amplitude and In-Phase Quadrature
- Let source 1 be retarded by 45° and source 2 advanced by 45 °.
- If the special case of $d = \lambda/4$, (9) becomes



$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right) \qquad (11)$$

It's a cardioid-shaped, unidirectional pattern with maximum field in the negative x direction. The space pattern is figure-of-revolution of this pattern around x axis.

4. Arrays of two isotropic point sources

- Case 4. General case of two isotropic point sources of equal amplitude and any phase difference
- Let us consider the any phase difference δ.
- The total phase difference ψ between the fields from source 2 and source 1 at a distant point in the direction ϕ is then

$$\psi = d_r \cos\phi + \delta \qquad (11)$$

- Taking source 1 as the reference for phase, the positive sign indicates that source 2 is advanced in phase by the angle δ. A minus would be used to indicate a phase retardation.
- If it is referred to the centerpoint of the array the phase of the field from source 1 at a distant point is given by - ψ/2, and that from source 2 by + ψ/2. The total field is then,

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2} = E_0 \cos\frac{\psi}{2} \qquad (12)$$

- Normalizing it, we have the general expression for the field pattern of two isotropic sources of equal amplitude and arbitrary phase,

$$E = \cos\frac{\psi}{2} \qquad (13)$$

5. Non-isotropic but similar point sources and the principle of pattern multiplication

- The word similar is here used to indicate that the variation with absolute angle ϕ of both the amplitude and phase of the field is the same.

- The maximum amplitudes of the individual sources may be unequal. If, however, they are also equal, the sources are not only similar but are *identical*.

- As an example, let us reconsider Case4 in section4, in which the sources are identical, with the modification that both sources 1 and 2 have field patterns given by

$$E_0 = E_0' \sin\phi$$

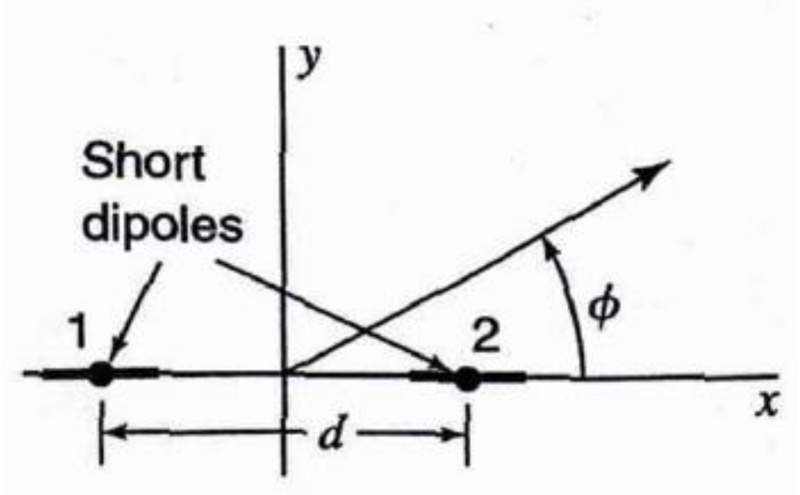
- Substituting it to equation (12) and normalizing, it gives the field pattern of the array as

$$E = \sin\phi \cos\frac{\psi}{2}$$

$$where\ \psi = d_r \cos\phi + \delta$$

5. Non-isotropic but similar point sources and the principle of pattern multiplication

- Patterns of this type might be produced by short dipoles oriented parallel to the x axis as suggested in the following figure.



- This results is the same as obtained by multiplying the pattern of the individual source (sinϕ) by the pattern of the two isotropic point source (cos ψ/2).

5. The principle of pattern multiplication

- *The field pattern of an array of nonisotropic but similar point sources is the product of the pattern of the individual source and the pattern of an array of isotropic point sources having the same locations, relative amplitudes, and phase as the nonisotropic point sources .*

The principle may be applied to arrays of any number of sources provided only that they are similar.

The individual nonisotropic source or antenna may be of finite size but can be considered as a point source situated at the point in the antenna to which phase is referred. This point is said to be the *phase center.*

The above discussion of the pattern multiplication has been concerned only with the field pattern or magnitude of the field. If the field of the nonisotropic source and the array of isotropic sources vary in phase with space angle, i.e., have a phase pattern which is not a constant, the statement may be extended to include this more general case as follows.

- ***The total field pattern of an array of nonisotropic but similar source*** *is the product of the individual source pattern and the pattern of an array of isotropic sources each located at the phase center of the individual source and having the same relative amplitude and phase, while total phase pattern is the sum of the phase patterns of the individual source and the array of isotropic point sources.*

5. The principle of pattern multiplication

- The total phase pattern is referred to the phase center of the array. In symbols, the total field E is then

$$E = f(\theta,\phi)F\,(\theta,\phi)/f_P\,(\theta,\phi) + F_P(\theta,\phi)$$

- Where

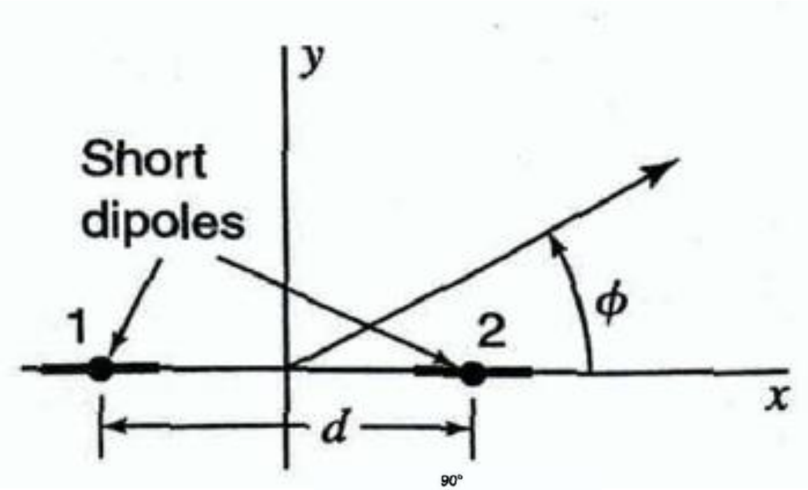
$f(\theta,\phi)$=field pattern of individual source
$f_P(\theta,\phi)$=phase pattern ofindividual source
$F(\theta,\phi)$=field pattern of array of isotropic sources
$F_P(\theta,\phi)$=phase pattern array of isotropic sources

- The patterns are expressed in above equation as a function of both polar angles to indicate that the principle of pattern multiplication applies to space patterns as well as to the two-dimensional cases we have been considering.

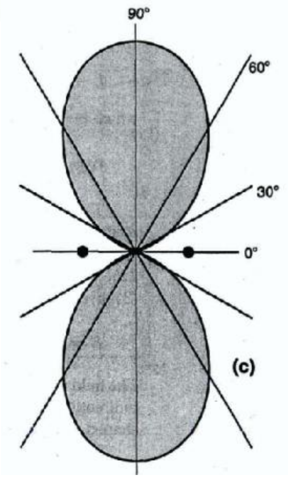
5. The principle of pattern multiplication

- Case 1. Assume two identical point source separated by a distance d, each source having a field pattern given by $E = E_0' \sin\varphi$ as might be obtained by two short dipoles. Let $d=\lambda/2$, and the phase angle δ=0. then the total field pattern is

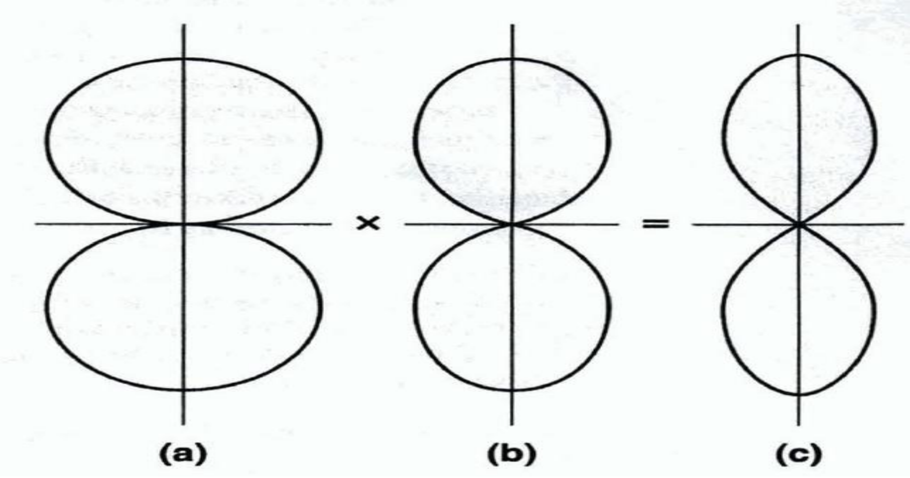
$$E = \sin\phi \cos\left(\frac{\pi}{2}\cos\phi\right)$$



- T … ving figures, as the product of the individual source pattern sh … shown at (b). The pattern is sharper than it was in case1 of se … is instance, the maximum field of the individual source is in th … vith the direction of the maximum field for the array of two is

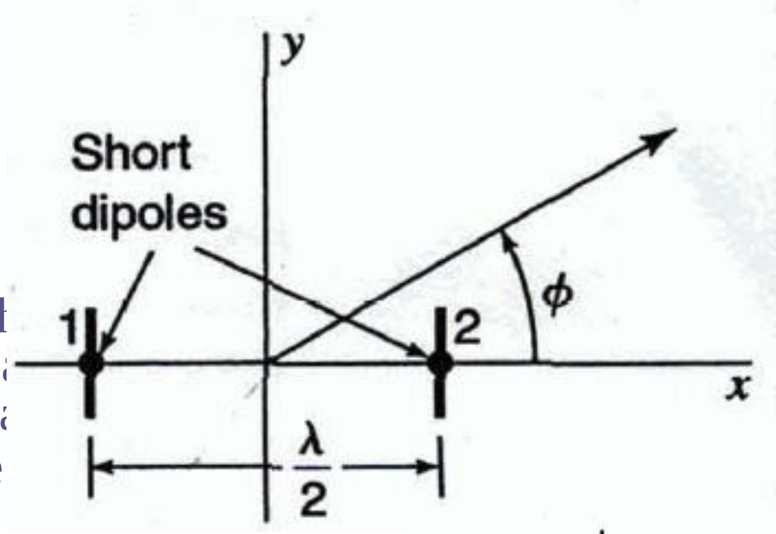


case1 of section4



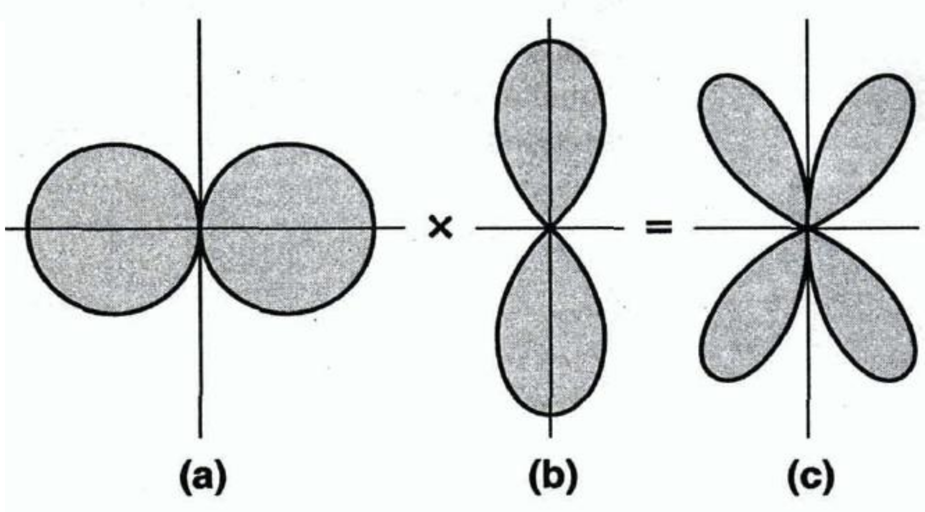
5. The principle of pattern multiplication

- Case 2. Assume two identical point source separated by a distance *d*, each source having a field pattern given by $E = E'_0 cos\varphi$ as might be obtained by two short dipoles oriented parallel to the y axis. Let $d=\lambda/2$, and the phase angle δ=0. then the total normalized field pattern is



$$E = cos\,\phi\, sin\left(\frac{\pi}{2}\cos\phi\right)$$

- In th … individual source is in the direction(ϕ=0°) of a null from the a … a null in the direction (ϕ=90°) of the pattern maximum of the a …
- The … obes with nulls at the x and y axes.



5. The principle of pattern multiplication

- The above examples illustrate two applications of the principle of pattern multiplication to arrays in which the source has a simple pattern.

- However, in more general case the individual source may represent an antenna of any complexity provided that the amplitude and phase of its field can be expressed as a function of angle, that is to say, provided that the field pattern and the phase pattern with respect to the phase center are known.

- If only the total field pattern is desired, phase patterns need not be known provided that the individual sources are identical.

- If the arrays in the above examples are parts of still larger arrays, the smaller arrays may be regarded as nonisotropic point sources in the larger array– another application of the principle of pattern multiplication yielding the complete pattern.

- In this way, the principle of pattern multiplication can be applied n times to find the patterns of arrays of arrays of arrays.

- Conclusions

1. The radiation density and radiation intensity of a point source.
2. The directivities of the typical point source patterns.
3. Arrays of two isotropic point source of same amplitude and in/out phases, and their field patterns.

- Questions
  - How to calculate the directivity of a point source with Sine (Doughnut) power pattern?
  - The principle of pattern multiplication.
  - Pattern synthesis by pattern multiplication.

Thank you!